Ed [illegible]

Morning Session

B1

N employees, $e_i = \begin{cases} 1 & \text{work} \\ 0 & \text{shirk} \end{cases}$, $c(e_i) = e_i$

$y_i = \begin{cases} z \text{ s.t. } 1 < z < N & \text{if work} \\ 0 & \text{if shirk} \end{cases}$

Firm output $= \sum y_i$

Shares are such that $s_i > 0$, $\sum s_i = 1$

$u_i = s_i \left(\sum_i y_i \right) - c(e_i)$

Solving FB:

Welfare $= \sum_i s_i \left(\sum_i y_i \right) - \sum c(e_i) \Rightarrow$ max welfare needs $e_i = 1 \; \forall i$

Then $\sum_i s_i (Nz) - N = Nz - N = N(z-1)$

$u_i^{FB} = \frac{1}{N} \cdot N(z-1) = z - 1 > 0$ as $1 < z < N$

Solving the SB:

Assuming an equal shares contract, $s_i = \frac{1}{N}$

$u_i = s_i \left(\sum y_i \right) - e_i = \frac{\sum y_i}{N} - e_i = \frac{\sum_{-i} y_i + z}{N} - 1$ if work

and $\frac{\sum_{-i} y_i}{N}$ if shirk.

Ed Egan
Morning Session
B1

If everyone else shirks $\sum_{-i} y_i = 0 \quad \therefore u(\text{shirk}) = 0$

$u(\text{work}) = \frac{Z}{N} - 1 < 0 \quad \therefore$ shirk.

But consider m people work $(m > 1)$

Then $u(\text{shirk}) = \frac{mZ}{N}$

$u(\text{work}) = \frac{(m+1)Z}{N} - 1 = \frac{mZ}{N} + \frac{Z}{N} - 1 < u(\text{shirk})$

$\therefore$ Dominant strategy to shirk with equal shares.

But consider: Q people get $s_i = \varepsilon$ $\quad \varepsilon > 0$ but arbitrarily small
Remaining people get equal shares:

The "shirk point" is $u_i(e_i = 0 \mid m-1 \text{ work}) \geq u_i(e_i = 1 \mid m \text{ work})$

$$\therefore \quad \frac{1-Q\varepsilon}{N-Q} \cdot (m-1)z \geq \frac{1-Q\varepsilon}{N-Q} \cdot mz - 1$$

$$-\frac{z(1-Q\varepsilon)}{N-Q} \geq -1$$

$$z(1-Q\varepsilon) \leq N-Q$$

Letting $\varepsilon \to 0$ this gives a "shirk point" of $z \leq N-Q \quad \forall m$

$\therefore$ with $z \geq N-Q$ all will work

$\therefore$ set $Q \geq N-z$ such that Q is the smallest integer

Ed Egan
Morning Session
B)

... that satisfies this equality

The output will be $z(N-Q)$ and this will be the optimum contract — each agent's work payoff will be greater than or equal to their shirk payoff and the outside option is Wendy's $(U_r = 0)$. — Note: equal shares for working members is best because they are identical.

2.) For Carol to credibly commit to burning some out I assume she must be the residual claimant and therefore can not be a productive team member.

Carol offers a contract like this to the N-1 other team members

$$w_i = \begin{cases} \frac{1}{N-1} \cdot (N-1)(z) & \text{if } \sum_i y_i \geq (N-1)(z) \\ 0 & \text{if } \sum_i y_i < (N-1)(z) \end{cases}$$

This would be better than the above contract only if

$$Q > 1 \implies N - Z > 1 \implies Z < N-1$$

Note that I have ignored Carol's two-sided moral hazard problem as per Eswaran & Kotwal (1984) for the reasons in Andolfatto & Nosal (1997).

Ed Egan
Morning Session
B1

3.) Supposing CARA preferences:

$$u(I) = -e^{-\rho I}$$

I would use the "decimation" contract of Rasmussen (1987). That is one agent would be selected at random to get the full payout, and all other team members would get zero if a performance target was not met.

For the agents to take the contract the expected payout in ~~[illegible]~~ in the "good" state (where they meet the target) must be greater than their outside option.

To get the punishment big enough the expected payout to a single defection must be negative. (≤ 0).

$\therefore$ the contract is:

$$b_i = \begin{cases} \frac{1}{N} \cdot N(Z) & \text{if } \sum_i y_i \geq nZ \\ \frac{1}{N} \cdot N(Z) & \text{if } \sum_i y_i < nZ \text{ with probality } \frac{1}{N} \\ 0 & \text{with probability } \frac{N-1}{N} \end{cases}$$

Expected payout in the bad state is therefore $\frac{Z}{N} < 1$ — even if the agent exerts no effort

$\therefore$ for any $\rho > 0$ this contract will work