1 Dixit & Stiglitz's model of monopolistic competition

Objective:

* introduce model of product differentiation driven by a sheer taste for variety (not out of risk diversification or distance)

* study market equilibrium and entry

* ask whether markets will provide too much or too little entry

  – business stealing forces tend to create too much entry

  – but perhaps that result is not robust to other circumstances

* steps: 1. consumer optimization; 2. firm optimization of production scale and entry decisions; 3. solving for number of firms; 4. comparison to planner solution

n firms (n large) producing respective differentiated goods $x_1, x_2, ... x_n$ which sell for $p_1, p_2, ..., p_n$

Good x_0 is the numeraire

Representative consumer has preferences

$$U = u\left(x_0, \left(\sum_{i=1}^{n} x^{\rho}\right)^{\frac{1}{\rho}}\right),$$

and budget $B = x_0 + \sum_{i=1}^{n} p_i x_i$

Consumer maximizes utility given prices

Free entry (so zero profits in equilibrium)

1. Consumer behavior

Using budget constraint, express consumer's problem as,

$$\max_{x_i} u\left(B - \sum_{i=1}^{n} p_i x_i, \underbrace{\left(\sum_{i=1}^{n} x^{\rho}\right)^{\frac{1}{\rho}}}_{y}\right)$$

Foc i:

$$-p_i u_{x_0} + u_y \frac{1}{\rho}\left(\sum_{i=1}^{n} x_i^{\rho}\right)^{\frac{1}{\rho}-1} \rho x_i^{\rho-1} = 0, \; i = 1, 2, ..., n$$

or

$$
\begin{aligned}
x_i^{1-\rho} &= \frac{1}{p_i} \frac{\left(\sum_{i=1}^n x_i^\rho\right)^{\frac{1-\rho}{\rho}} u_y}{u_{x_0}} \\
x_i &= \left(\frac{1}{p_i}\right)^{\frac{1}{1-\rho}} \left(\frac{u_y}{u_{x_0}}\right)^{\frac{1}{1-\rho}} \left(\sum_{i=1}^n x_i^\rho\right)^{\frac{1}{\rho}} = \left(\frac{1}{p_i}\right)^{\frac{1}{1-\rho}} q^{\frac{1}{1-\rho}} y,
\end{aligned}
$$

not an explicit solution (eq 7 in paper).

2. Market behavior

Note problem for a firm: changing own production p_i will directly reduce demand x_i but will also trigger changes in demands for (and prices of) other firms, affecting elements q and y.

But if we could consider q invariant in firm's decisions on x_i and p_i, then demand elasticity facing firm is easy to characterize. To confirm, note,

$$\frac{dx_i}{dp_i} = -\frac{1}{1-\rho}\left(\frac{1}{p_i}\right)^{\frac{1}{1-\rho}}\frac{1}{p_i}q^{\frac{1}{1-\rho}}y + \left(\frac{1}{p_i}\right)^{\frac{1}{1-\rho}}\frac{1}{1-\rho}q^{\frac{1}{1-\rho}-1}\frac{dq}{dp_i}y + \left(\frac{1}{p_i}\right)^{\frac{1}{1-\rho}}q^{\frac{1}{1-\rho}}\frac{dy}{dp_i},$$

so if $\frac{dq}{dp_i}, \frac{dy}{dp_i} \simeq 0$, we have,

$$\frac{dx_i}{dp_i} = -\frac{1}{1-\rho}\left(\frac{q}{p_i}\right)^{\frac{1}{1-\rho}}\frac{y}{p_i},$$

Thus, elasticity of demand facing firm i is

$$\frac{dx_i}{dp_i}\frac{p_i}{x_i} = -\frac{1}{1-\rho}\left(\frac{q}{p_i}\right)^{\frac{1}{1-\rho}}\frac{y}{x_i} = -\frac{1}{1-\rho}\left(\frac{q}{p_i}\right)^{\frac{1}{1-\rho}}\frac{y}{\left(\frac{q}{p_i}\right)^{\frac{1}{1-\rho}}y} =$$

$$\frac{dx_i}{dp_i}\frac{p_i}{x_i} = -\frac{1}{1-\rho}.$$

Can we consider $\frac{dq}{dp_i} \simeq 0$?

Yes if n is large. Changes by one firm have negligible aggregate effects

q is defined as $\left(\sum_{i=1}^{n} p_i^{-\frac{1}{\beta}}\right)^{-\beta}$, $\beta = \frac{1-\rho}{\rho}$.

$$\frac{dq}{dp_i} = \left(\sum_{i=1}^{n} p_i^{-\frac{1}{\beta}}\right)^{-\beta-1} p_i^{-\frac{1}{\beta}-1}$$

$$\frac{dq}{dp_i}\frac{p_i}{q} = \frac{p_i^{-\frac{1}{\beta}}}{q^{-\frac{1}{\beta}}} = \left(\frac{q}{p_i}\right)^{\frac{1}{\beta}}$$

Now note that in symmetric equilibrium, $y = \left(\sum_{i=1}^{n} x_i^{\rho}\right)^{\frac{1}{\rho}} = n^{\frac{1}{\rho}} x, q = n^{-\beta} p$, so

$$\frac{dq}{dp_i}\frac{p_i}{q} = \left(\frac{q}{p_i}\right)^{\frac{1}{\beta}} = \left(\frac{1}{n^{\beta}}\right)^{\frac{1}{\beta}},$$

which goes to zero as $n \to \infty$, so firms behaving as if in partial equilibrium is warranted, and we can consider demand of the form

$$x_i = k p_i^{-\frac{1}{1-\rho}}.$$

However, equilibrium market demand (Chamberlinian demand) is less elastic because of general equilibrium effects.

Firms solve

$$\max_{p_i} (p_i - c)\, k p_i^{-\frac{1}{1-\rho}} - f$$

Foc is,

$$\begin{aligned} k p_i^{-\frac{1}{1-\rho}} - \frac{1}{1-\rho}(p_i - c)\, k p_i^{-\frac{1}{1-\rho}-1} &= 0 \\ \frac{1}{1-\rho}\frac{(p_i - c)}{p_i} &= 1 \\ p_i^* &= \frac{c}{\rho} \end{aligned}$$

In a free entry equilibrium, we must have zero profits:

$$\begin{aligned} (p-c)\,x &= f \\ \left(\frac{c}{\rho}-c\right)x &= f \\ x^{*} &= \frac{f}{c}\frac{\rho}{(1-\rho)}. \end{aligned}$$

3. Solving for number of firms

Foc of consumer was,

$$
\begin{aligned}
u_y \frac{1}{\rho} \left(\sum_{i=1}^{n} x_i^{\rho} \right)^{\frac{1}{\rho}-1} \rho x_i^{\rho-1} &= p^* u_{x_0} \\
u_y n^{\frac{1-\rho}{\rho}} x^{*1-\rho} x^{*\rho-1} &= \frac{c}{\rho} u_{x_0} \\
n^{*\frac{1-\rho}{\rho}} &= \frac{c}{\rho} \frac{u_{x_0}}{u_y},
\end{aligned}
$$

where $\frac{u_{x_0}}{u_y}$ is a function of n^*, x^* and parameters.

We would like to compare this solution to that which would emerge from a planner's program.

4. Planner would use marginal cost pricing and cover fixed costs through transfers financed with lump sum taxation

So planner would solve,

$$\max_x u\left(B - nf - ncx, xn^{\frac{1}{\rho}}\right)$$

and then optimize number of firms. But by virtue of envelope theorem, we might as well perform both optimizations simultaneously,

Foc x and Foc n are, respectively,

$$\begin{aligned} -ncu_{x_0} + n^{\frac{1}{\rho}} u_y &= 0 \\ (-f - cx)\, u_{x_0} + \frac{1}{\rho} x n^{\frac{1}{\rho}-1} u_y &= 0, \end{aligned}$$

yielding planner solutions (x^p, n^p).

4. To compare market and planner solutions use more specific example:

$$U = x_0^{\alpha} \left(\sum_{i=1}^{n} x_i^{\rho} \right)^{\frac{1}{\rho}}.$$

Recall market solution n^*:

$$n^* = \left(\frac{c}{\rho} \frac{u_{x_0}}{u_y} \right)^{\frac{\rho}{1-\rho}},$$

and note that

$$\begin{aligned} u_{x_0} &= \alpha x_0^{\alpha-1} \left(\sum_{i=1}^{n} x_i^{\rho} \right)^{\frac{1}{\rho}} \\ u_y &= x_0^{\alpha}, \end{aligned}$$

so we can write,

$$n^* = \left(\frac{c}{\rho}\right)^{\frac{\rho}{1-\rho}} \left(\frac{\alpha x_0^{\alpha-1}\left(\sum_{i=1}^{n} x_i^{\rho}\right)^{\frac{1}{\rho}}}{x_0^{\alpha}}\right)^{\frac{\rho}{1-\rho}}$$

$$n^{*\frac{1-\rho}{\rho}} = \frac{c}{\rho}\frac{\alpha n^{*\frac{1}{\rho}} x^*}{B - n^* p^* x^*},$$

and some more algebra leads to the solution...

$$
\begin{aligned}
{n^*}^{\frac{1-\rho}{\rho}}\left(B - n^* p^* x^*\right) &= \frac{c}{\rho}\alpha {n^*}^{\frac{1}{\rho}} x^* \\
B{n^*}^{\frac{1-\rho}{\rho}} - {n^*}^{\frac{1-\rho}{\rho}+1} p^* x^* &= \frac{c}{\rho}\alpha {n^*}^{\frac{1}{\rho}} x^* \\
B{n^*}^{\frac{1-\rho}{\rho}} &= {n^*}^{\frac{1}{\rho}} p^* x^* \left(1+\alpha\right) \\
{n^*}^{\frac{1}{\rho}-1} &= {n^*}^{\frac{1}{\rho}} \frac{p^* x^* \left(1+\alpha\right)}{B} \\
n^* &= \frac{B}{p^* x^* \left(1+\alpha\right)} = \frac{B}{\frac{c}{\rho}\frac{f}{c}\frac{\rho}{(1-\rho)}\left(1+\alpha\right)} \\
n^* &= \frac{B}{\frac{f}{(1-\rho)}\left(1+\alpha\right)}
\end{aligned}
$$

Now turn to the example's planner's solution to complete the comparison.

The planners Focs were,

$$\begin{aligned} cu_{x_0} &= n^{\frac{1}{\rho}-1} u_y \\ (f + cx)\, u_{x_0} &= \frac{x}{\rho} n^{\frac{1-\rho}{\rho}} u_y \end{aligned}$$

and using the first equation into the second we get,

$$\begin{aligned} \frac{n^{\frac{1-\rho}{\rho}}}{c} &= \frac{u_{x_0}}{u_y} \\ (f + cx)\, \frac{n^{\frac{1-\rho}{\rho}}}{c} &= \frac{x}{\rho} n^{\frac{1-\rho}{\rho}} \end{aligned}$$

leaving,

$$\begin{aligned} \frac{f}{c} + x - \frac{x}{\rho} &= 0 \\ x^p &= \frac{f}{c}\frac{\rho}{1-\rho} = x^*. \end{aligned}$$

The planner chooses the same quantities!

Then, using example utility specification in first planner Foc,

$$
\begin{aligned}
n^{\frac{1-\rho}{\rho}} &= c\frac{u_{x_0}}{u_y} = c\frac{\alpha x_0^{\alpha-1}\left(n^{\frac{1}{\rho}}x\right)}{x_0^{\alpha}} \\
n^{\frac{1-\rho}{\rho}} &= c\frac{\alpha\left(n^{\frac{1}{\rho}}x\right)}{x_0} \\
n^{\frac{1-\rho}{\rho}} &= c\frac{\alpha\left(n^{\frac{1}{\rho}}x\right)}{B-nf-ncx} \\
n^{\frac{1-\rho}{\rho}}(B-nf) - n^{\frac{1}{\rho}}cx &= c\alpha n^{\frac{1}{\rho}}x \\
n^{\frac{1-\rho}{\rho}}(B-nf) &= n^{\frac{1}{\rho}}cx\,(1+\alpha) \\
B-nf &= ncx\,(1+\alpha) \\
B &= n\,(cx\,(1+\alpha)+f) \\
n^{p} &= \frac{B}{\left((1+\alpha)\frac{\rho}{1-\rho}+1\right)f}
\end{aligned}
$$

So is it true the market will generate too much entry?

$$n^* = \frac{B}{\frac{f}{(1-\rho)}(1+\alpha)} > \frac{B}{\left((1+\alpha)\frac{\rho}{1-\rho}+1\right)f} = n^p?$$

We would need,

$$\begin{aligned}
\frac{\left((1+\alpha)\frac{\rho}{1-\rho}+1\right)f}{\frac{f}{(1-\rho)}(1+\alpha)} &> 1 \\
\frac{\left((1+\alpha)\frac{\rho}{1-\rho}+1\right)(1-\rho)}{(1+\alpha)} &> 1 \\
\rho + \frac{(1-\rho)}{1+\alpha} &> 1,
\end{aligned}$$

$$\begin{aligned}\frac{(1-\rho)}{1+\alpha} &> 1-\rho \\ 1 &> 1+\alpha,\end{aligned}$$

a contradiction. The example illustrates the Dixit-Stiglitz argument that the market may not create too much entry.

Intuition: business stealing effect mitigated by difficulties in appropriating consumer surplus.