FW: Answer to Field Exam Question 2007/2008 B2 - Opportunistic Sellers

Tarek Ghani <tarek_ghani@haas.berkeley.edu> **Tue, Apr 5, 2011 at 9:21 AM**
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Question B.2: Opportunistic sellers

Consider an economy with many identical Buyers that can each engage in a transaction with one of many sellers. For concreteness, imagine that there are more buyers than sellers and that the market for transactions must clear. The transaction can either succeed or fail. Each buyer's value of "success" is 1 and of "failure" is 0.

There are two kinds of Sellers. A proportion are "good" and they succeed with probability $p > 0$. A proportion β are "opportunistic" and can choose some effort, $e \in [0, 1]$ at a personal cost of $c(e)$ where $c'(0) = 0, c'(1) = \infty$ and $c''(e) > 0 \ \forall e = 0$. The opportunistic types succeed with probability ep.

The economy operates for 2 periods. Sellers live for two periods but a new cohort of buyers is active in each period, and second period buyers can observe the first period outcome of transactions.

Effort and types are not observable to buyers so that in each period buyers who wish to transact with sellers will pay a price for the transaction in advance, after which the service/good is delivered and failure/success is observed.

a.) What level of e would maximize total surplus?

b.) An equilibrium is defined by a price w_1 that buyers pay sellers in the first period, and history contingent prices $w_2(S)$ and $w_2(F)$ (for Success and Failure respectively) that buyers pay sellers in the second period, so that expectations about outcomes are correct, and sellers best respond to current and future outcomes. Is there an equilibrium where opportunistic sellers exert the level of e you found in (a) in each period? If so, show it. If not, explain why not.

c.) Is there an equilibrium in which opportunistic sellers choose no effort in both periods? If so, show it. If not, explain why not.

d.) Find the equilibrium of this market, and argue that it is unique.

e.) If the sellers exert effort in the equilibrium you found in (d) in some period, what market mechanism provides them with incentives? How would you interpret his?

From: Orie Shelef
Sent: Friday, April 01, 2011 10:58 PM
To: Tarek Ghani
Subject: RE: Answer to Field Exam Question - Opportunistic Sellers

Here's my exam answer:

2007 & 2008 (B2) – Opportunistic Sellers

Without Loss, Assume no discounting.

a) Let e be the vector (e_1, e_2) of effort choices by the opportunistic seller.

Effort only influences the welfare from a transaction with an opportunistic seller. In each period, effort e_i causes the opportunistic seller to pay a personal cost of effort $c(e_i)$, and causes the probability of a success, which has a value of 1 over a failure to the buyer, to be $e_i p$. So, first best effort must satisfy $c'(e_i) = p$, so $e^{FB} = \left(c'^{(-1)}(p), c'^{(-1)}(p)\right)$. Because of the assumptions on the cost function and $p > 0$, $c'^{(-1)}(p) \in (0,1)$

b) Given that they are paid in advance, and the second period is the final period, it is obvious that $e_2^* = 0$ in any equilibrium. Thus, there is no equilibrium where e^{FB} is exerted. But also, first best effort is not exerted in the 1st period.

Because there are more buyers than sellers, sellers are paid their expected value as believed by the market.

Let $u(S)$ and $u(F)$ be the beliefs about the probability that a seller is good having observed a history of a success and failure respectively. Let $\hat{e}$ be the market's belief about effort in period 1. Then by bayes rule:

$$u(S) = \frac{(1-\beta)p}{(1-\beta)p + \beta\hat{e}p}$$

$$u(F) = \frac{(1-\beta)(1-p)}{(1-\beta)(1-p) + \beta(1-\hat{e}p)}$$

Thus, since $e_2^* = 0$ we know that:

$$w_2(S) = u(S)p$$

$$w_2(F) = u(F)p$$

Now consider the optimization problem of an opportunistic seller

$$\max_e w_1 + c(e) + epw_2(S) + (1-ep)w_2(F)$$

Which gives the FOC:

$$c'(e) = pw_2(S) - pw_2(F) = p\Delta W_2$$

Where:

$\Delta W_2 = w_2(S) - w_2(F)$

This leads to first best effort iff $\Delta W_2 = 1$, which implies that $w_2(S) = w_2(F) + 1$

Which implies that utility for the buyer is 1 greater having seen a success than a failure. But, the maximum utility the buyer can receive in the second period is the utility of contracting with a good seller, which gives utility p. But, since wages are bounded by zero (otherwise sellers would reject them). $\Delta W_2 \leq p$. If $p < 1$ then we have a contradiction, and we do not get first best in the first period.

Suppose $p = 1$. Then, $w_2(F) = 0$ and $w_2(S) = \frac{1-\beta}{(1-\beta)+\beta\hat{e}}$. So we get $\Delta W_2 = 1$ iff $\hat{e} = 0$, which contradicts that e_1^* is first best.

There is no incentive for effort in the second period, and thus no effort in that period. Further, because opportunistic types exert no effort in the second period, if they exert any effort in the first period, the wage following a success must be lower p so they do not face first best incentives in the first period.

c) From above, we know:

$e_2^* = 0$ in any equilibrium and the maximization problem yields $c'(e) = p\Delta W_2$.

By assumption $c'(0) = 0$, so for $e_1^* = 0$ we need $\Delta W_2 \leq 0$ which implies:

$$u(S) \leq u(F)$$

$$\frac{(1-\beta)p}{(1-\beta)p + \beta\hat{e}p} \leq \frac{(1-\beta)(1-p)}{(1-\beta)(1-p) + \beta(1-\hat{e}p)}$$

If $e_1^* = 0$ then $\hat{e} = 0$, so

$$\frac{(1-\beta)p}{(1-\beta)p} \leq \frac{(1-\beta)(1-p)}{(1-\beta)(1-p) + \beta}$$

A contradiction, so $e_1^* \neq 0$. Because $\Delta W_2 > 0$ there is an incentive for opportunistic sellers to exert some effort in the first period. Since $c'(0) = 0$ they exert positive effort.

d) From above we know From above, we know:

$e_2^* = 0$ in any equilibrium and the maximization problem yields $c'(e) = p\Delta W_2$. We also know that $e_1 < 1$ from b and since $c'(1) = \infty$. So restricting to interior solutions:

$$c'(e) = p\Delta W_2$$

$$= p^2\big(u(S) - u(F)\big)$$

$$= p^2(1-\beta)\left[\frac{(1-\beta)p}{(1-\beta)p + \beta\hat{e}p} - \frac{(1-\beta)(1-p)}{(1-\beta)(1-p) + \beta(1-\hat{e}p)}\right]$$

Since, in equilibrium beliefs are correct, e_1^* is the solution to:

$$c'(e) = p^2(1-\beta)\left[\frac{(1-\beta)p}{(1-\beta)p+\beta ep} - \frac{(1-\beta)(1-p)}{(1-\beta)(1-p)+\beta(1-ep)}\right]$$

Existence follows since the LHS is continuous and takes on all values $\in [0,\infty)$ over the domain $e \in [0,1]$ and for all $\beta \in [0,1]$ and $p \in [0,1]$ the RHS is positive.

Uniqueness: It suffices to show that $p\Delta W_2$ is non-increasing on $e \in (0,1)$ since $c'(\cdot)$ is strictly increasing by assumption.

$$\frac{\partial p\Delta W_2}{\partial e} = p^2(1-\beta)\left[\frac{-p^2\beta}{((1-\beta)p+\beta ep)^2} - \frac{(1-p)\beta p}{((1-\beta)(1-p)+\beta(1-ep))^2}\right]$$

Since $e \in (0,1)$, $p \in (0,1]$, and $\beta \in [0,1]$

$$\frac{-p^2\beta}{((1-\beta)p+\beta ep)^2} \leq 0$$

$$\frac{(1-p)\beta p}{((1-\beta)(1-p)+\beta(1-ep))^2} \geq 0$$

So

$$\frac{\partial p\Delta W_2}{\partial e} \leq 0$$

Generating single crossing and thus uniqueness.

Formally, the equilibrium is a strategy for the opportunistic seller of e_1^* as the solution to $c'(e) = p\Delta W_2$, $e_2^* = 0$.

First period buyers' strategies of $w_1 = (1-\beta)p + \beta e_1^* p$.

Second period buyers' strategies $w_2(S) = \frac{(1-\beta)p^2}{(1-\beta)p+\beta e_1^* p}$, $w_2(F) = \frac{(1-\beta)(1-p)p}{(1-\beta)(1-p)+\beta(1-e_1^* p)}$ and beliefs $u(S) = \frac{(1-\beta)p}{(1-\beta)p+\beta e_1^* p}$, $u(F) = \frac{(1-\beta)(1-p)}{(1-\beta)(1-p)+\beta(1-e_1^* p)}$

e) Incentives are provided through the reputations of the opportunistic sellers. They exert effort in the first period to "pretend" to be good sellers. In the first period, they invest effort because it increases their reputation and allows them to get paid more in the second period. Thus they have career concerns. In this two period game, these career concerns provide some, but less than first best effort in the first period, and no effort in the final period.

66 attachments

e **image001.png**
1K

(e_1, e_2) **image002.png**
1K

e_i **image003.png**
1K

$c(e_i)$ **image004.png**
1K

$e_i p$ **image005.png**
1K

$c'(e_i) = p$ **image006.png**
1K

$e^{FB} = \left(c'^{(-1)}(p), c'^{(-1)}(p)\right).$ **image007.png**
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$p > 0$ **image008.png**
1K

$c'^{(-1)}(p) \in (0,1)$ **image009.png**
1K

$e_2^* = 0$ **image010.png**
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e^{FB} **image011.png**
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$u(S)$ **image012.png**
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$u(F)$ **image013.png**
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$\hat{e}$ **image014.png**
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$u(S) = \frac{(1-\beta)p}{(1-\beta)p + \beta\hat{e}p}$ **image015.png**
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$u(F) = \frac{(1-\beta)(1-p)}{(1-\beta)(1-p) + \beta(1-\hat{e}p)}$ **image016.png**
2K

$w_2(S) = u(S)p$ **image017.png**
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$w_2(F) = u(F)p$ **image018.png**
1K

$\max_e w_1 + c(e) + epw_2(S) + (1-ep)w_2(F)$ **image019.png**
2K

$c'(e) = pw_2(S) - pw_2(F) = p\Delta W_2$ **image020.png**
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