• effort is never observable

• normally, we contract on output, which is verifiable

Steve's Exam Question

a) We first note since cost is observable but not verifiable, we will have to rely on a relational contract to self enforce the firm's 2nd period payment to the manager. Since we are assuming the manager will make a take it or leave it offer to the firm, he will offer a contract that makes the firm indifferent between accepting and rejecting, maximizing rent.

if it depends on observed cost

Now if the firm, simply randomly selects its manager from the market (or facing pooling manager types), it expects the following cost: $p\overline{\beta} + (1-p)\underline{\beta} = p\Delta\beta + \underline{\beta} \equiv \mu_\beta$, assuming zero effort. Clearly, effort is zero the second period, since it is costly for the manager and there is no way for the firm to credibly compensate him for such effort.

Agent: Manager
Principal: Firm.

If the firm rejects the managers contract, it will then have to randomly select a manager for the final period (we assume both managers and firms are living 2 periods). Hence, the "breakup" payoff for the firm is cost μ_β. Hence, since C_1 is not verifiable, the second period payment of any contract contingent on C_1 cannot exceed μ_β. (otherwise firm will go to the market)

someone chosen randomly from the mkt

We first normalize the net benefit of a high cost manager exerting zero effort to a firm as zero (and thus net benefit to the firm is $\Delta\beta$ when using a low cost manager exerting no effort). We assume any net benefit can then be captured by the low cost manager via compensation. Thus consider the following contract proposed by the low cost manager:

[first period you'll get more than if you were pooling]

Period 1: $w_1 = p\Delta\beta + \delta p^2\Delta\beta$

Period 2: $\begin{cases} w_2 = p\Delta\beta \text{ if } C_1 = \underline{\beta} \\ w_2 = 0 \text{ o.w.} \end{cases}$

Cost of effort for high cost pretending to be low cost

Due to convexity $f(p \cdot b) < pb$

We note $\varphi(\Delta\beta) < \delta\Delta\beta$ (as we were given by assumption), $0 < p < 1$, and the convexity of $\varphi(\cdot)$ imply we have $\varphi(p\Delta\beta) < \delta p\Delta\beta$. This means it is too costly for the high cost type to imitate the low cost type; hence, we get separation. Meanwhile, the low cost type does not have to exert any (costly) effort to obtain his second period bonus of $p\Delta\beta$.

cost benefit => since benefit > cost => separation

It would seem the low cost type would then want to add some effort to the contract since it costs him less for a unit of effort than the value to the firm up to the first best of having $\varphi'(e) = 1$. However, again since C_1 is not verifiable the maximum the firm can credibly commit to pay the 2nd period is $p\Delta\beta$, the difference between keeping the low cost worker with zero effort for certain or having to go to the open market to randomly pick a manager for the 2nd period.

We didn't have this before

w_2: $p\Delta\beta$

The extra $\delta p^2\Delta\beta$ paid the first period is then the present value of the $p\Delta\beta$ the firm saves by not having to pay this to the high cost types (i.e., p of the population) in the second period.

$\delta p^2\Delta\beta$ Knowing he's a low cost, this is what firm will save

$p(p\Delta\beta)$

Hence, the low cost type is simply extracting the present value of his rent $p\Delta\beta$, which comes from the fact it is less costly for him to provide any level of cost to the firm than the high cost type. So the firm is just indifferent in going along with the low cost type's proposal. However, the low cost type now earns a rent of $p\Delta\beta$, and is in total better off by $\delta p^2\Delta\beta$ compared to pooling (i.e., all

present value of cost savings

I am not sure they're pooling

workers are paid $p\Delta\beta$ both periods). Meanwhile, for the high type, the cost exceeds the benefit to imitate the low cost type.

b) First we consider the intertemporal ICs for the high and low cost type:

$\delta\overline{w}_2 - \varphi(\overline{e}_1) \geq \delta\underline{w}_2 - \varphi(\underline{e}_1 + \Delta\beta)$ $\quad$ $\text{IC}_{\text{high cost}}$

$\delta\underline{w}_2 - \varphi(\underline{e}_1) \geq \delta\overline{w}_2 - \varphi(\overline{e}_1 - \Delta\beta)$ $\quad$ $\text{IC}_{\text{low cost}}$

These just require the high and low cost types to prefer their own "package" from the firm. Note to act like a low cost type a high cost must put in less effort to act like the low cost. Meanwhile, the low cost can act like the high cost type by putting in less (costly) effort.

First, we add the two ICs together to get: $-\varphi(\overline{e}_1) - \varphi(\underline{e}_1) > -\varphi(\underline{e}_1 + \Delta\beta) - \varphi(\overline{e}_1 - \Delta\beta)$. We get the strict inequality from assuming strict convexity of our cost function (which we have when we use the below quadratic cost function).

Now consider the case of $\varphi(x) = x^2$. This then gives us: So?

$-(\overline{e}_1)^2 - (\underline{e}_1)^2 > (\underline{e}_1 + \Delta\beta)^2 + (\overline{e}_1 - \Delta\beta)^2 \Rightarrow -(\overline{e}_1)^2 - (\underline{e}_1)^2 > -(\overline{e}_1)^2 - (\underline{e}_1)^2 - 2\Delta\beta^2 + 2\Delta\beta\overline{e}_1 - 2\Delta\beta\underline{e}_1 \Rightarrow \underline{e}_1 - \overline{e}_1 + 2\Delta\beta > 0$

Hence, as $\Delta\beta \to 0$, we get $\underline{e}_1 - \overline{e}_1 > 0$.

Now note that $\overline{C} - \underline{C} = \Delta\beta + \underline{e}_1 - \overline{e}_1$. Hence, since with our ICs, as $\Delta\beta \to 0$, we have $\underline{e}_1 - \overline{e}_1 > 0 \Rightarrow \overline{C} - \underline{C} \not\to 0$ as $\Delta\beta \to 0$.

c) The optimal scheme as $\Delta\beta$ gets small is then to have managers pool because as $\Delta\beta \to 0$ we still require two different effort levels for each type (to maintain our ICs). However, since the productivity parameters are roughly the same now for both types, beacuse of the (strict) convexity of the cost function, there has to be a single preferred effort level. Hence, pooling implements such single effort level.