ECONOMETRICA
JOURNAL OF THE ECONOMETRIC SOCIETY

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Source: *Econometrica*, Vol. 56, No. 4 (Jul., 1988), pp. 755-785
Published by: The Econometric Society
Stable URL: http://www.jstor.org/stable/1912698

Econometrica, Vol. 56, No. 4 (July, 1988), 755–785

INCOMPLETE CONTRACTS AND RENEGOTIATION

BY OLIVER HART AND JOHN MOORE[1]

When drawing up a contract, it is often impracticable for the parties to specify all the relevant contingencies. In particular, they may be unable to describe the states of the world in enough detail that an outsider (the courts) could later verify which state had occurred, and so the contract will be incomplete. The parties can make up for this incompleteness to some extent by building into the contract a mechanism for revising the terms of trade as they each receive information about benefits and costs. One striking conclusion of our analysis is that because the parties can rescind the original contract and write a new one, severe limitations are placed on the form the revisions can take. Moreover, these limitations depend crucially on what means of communication the parties have at their disposal during the revision process.

We characterize an optimal contract in two cases. First, when a contract is being used to facilitate trade between two agents who must undertake relationship-specific investments, it is generally not possible to implement the first-best. For a particular example, we are able to confirm the idea that the second-best outcome will involve under-investment. Second, when a contract is being used to share risk, and there are no specific investments, we find that it is possible to implement the first-best provided messages sent between the agents can be publicly verified.

KEYWORDS: Incomplete contracts, renegotiation, specific investment.

1. INTRODUCTION

A PRINCIPAL FUNCTION of a long-term contract is to facilitate trade between two parties who must make relationship-specific investments. Once the investments have been sunk and the parties have become locked-in to each other, outside competition will have little impact on the terms of their trading, and so these must be governed instead by contractual provision. The difficult task facing the drafters of a contract is to anticipate and deal appropriately with the many contingencies which may arise during the course of their trading relationship. Since it may be prohibitively costly to specify, in a way that can be enforced, the precise actions that each party should take in every conceivable eventuality, the parties are in practice likely to end up writing a highly incomplete contract.

Problems of incomplete contracting have for some time been recognized as having important implications for the efficiency of long-term economic relationships, as well as providing possible explanations for the emergence of certain types of institution, such as the firm (see, e.g., Williamson (1985) and Klein, Crawford, and Alchian (1978)). Furthermore, it is clear from even a cursory

[1]We have benefited from the comments of a large number of people, and have received useful suggestions from seminar audiences at the University of Bonn, Boston University, University of Chicago, Harvard-MIT, LSE, Northwestern University, and Princeton University. We would particularly like to acknowledge very helpful discussions with Peter Cramton, Sandy Grossman, Eric Maskin, and Steven Matthews, and to thank a co-editor and two anonymous referees for their detailed reports on previous versions of the paper. Financial support from the Suntory Toyota International Centre for Economics and Related Disciplines at the LSE and, in the case of the first author, also from the University of Pennsylvania and the National Science Foundation is gratefully acknowledged.

glance at case law that most contractual disputes which come before the courts concern a matter of incompleteness (see, e.g., Dawson, Harvey, and Henderson (1982)). In spite of this, problems of incomplete contracting have received relatively little attention, presumably because of the difficulty of formalizing the costs of writing a contingent contract.

Note that a distinction can be drawn between problems arising from contractual incompleteness and those arising from asymmetries of information, although the overlap between the two is considerable. In the latter case, certain contingent statements are infeasible because the state of the world is not observed by all parties to the contract. In the case of contractual incompleteness, on the other hand, the parties may have the same information; what prevents the use of a complete contingent contract is the cost of processing and using this information in such a way that the appropriate contingent statements can be included and implemented. These "transactions costs" may also limit the complexity of contracts.

In this paper, our approach to developing a theory of incomplete contracts is to focus on the cost of writing a contingent clause in a sufficiently clear and unambiguous way that it can be enforced. Suppose that the states of the world ω are highly complex and of high dimension; ω may include the state of demand, what other firms in the industry are doing, the state of technology, etc. Many of these components may be quite nebulous. To describe ω in sufficient detail that an outsider (the courts) can verify whether a particular state $\omega = \hat{\omega}$ has occurred, and so enforce a contract which is contingent on ω, may be prohibitively costly. Under these conditions the contract will have to omit some (and in extreme cases, all) references to the underlying state.[2]

In spite of the incompleteness of the initial contract, it may be possible for the parties to revise and/or renegotiate the contract once ω is realized. This possibility is a principal concern of the present paper.[3] We shall analyze the form of an optimal contract under the assumption that the parties *always* have the option to renegotiate it later on.

Given rational expectations by the parties, the fact that revisions and/or renegotiation will occur will affect the form of the original contract. Less obvious, perhaps, is the fact that it will be in the interest of the parties to try to *constrain* in the original contract the final outcome of the revision/renegotiation process. That is to say, the parties face the problem of designing an optimal revision game to be played once ω is realized in order to yield final quantities and prices which are appropriately sensitive to the parties' benefits and costs. This game or mechanism design problem will be the focus of much of the paper.

[2] There are some previous papers that analyze incomplete contracts; see, e.g., Crawford (1988), Dye (1985), Grout (1984), Hall and Lazear (1984), Rogerson (1984), Shavell (1980), and Weitzman (1981). Most of these papers assume that certain contingent statements have a fixed cost associated with them —in the extreme case this cost is infinite and so the contingent statements cannot be included in the contract at all. An approach similar to the one taken here may, however, be found in the work of Bull (1987) and Grossman-Hart (1986, 1987).

[3] Revisions have also been studied by Rogerson (1984) and Shavell (1984).

In carrying out our analysis, we ignore all other transactions costs which may lead to incompleteness. One class of costs comes under the heading of bounded rationality. If the parties are boundedly rational, they may be unable to anticipate every eventuality, and may find it too difficult to decide (and reach agreement about) how to deal with all the eventualities which they do foresee (for a discussion of bounded rationality, see Simon (1981)). Bounded rationality may also limit the types and complexity of revision games that the buyer and seller can conceive of. We ignore the bounded rationality issue, not because we think it is unimportant, but because of the great difficulty of analyzing it formally. We also feel that in at least some situations, the parties to a contract may be sufficiently sophisticated to conceive of the relevant states of the world and to consider the types of revision processes we study, i.e., it is the inability to *describe* ω which really constitutes the major "transaction cost."

The paper is organized as follows. The model is set out in the next section, and the critical assumptions concerning the timing and transmission of messages are discussed. In Section 3, the class of possible trading prices is found when messages cannot be verified by outsiders. Section 4 does the same, but under the assumption that messages can be verified. These results are applied in Section 5. Section 6 contains conclusions.

2. THE MODEL

We consider the relationship between a buyer and a seller of a homogeneous good. The buyer and seller, who for most of our analysis are supposed to be risk neutral, write a contract at some initial date 0 which specifies the conditions of trade between them in the future.[4] To simplify, we assume that all trade occurs at a single date, date 2. At date 2, either one unit is traded, or zero. The buyer's valuation of one unit at date 2 is given by the random variable v and the seller's cost by the random variable c; the values of v and c are realized some time after date 0 but before date 2; at date 1, say.

After signing the contract at date 0, but before date 1, the buyer and seller make specific investments, β and σ respectively, which affect the distributions of v and c. We will assume that β and σ are sufficiently complex that it would be prohibitively costly to describe them in such a way that an outside court could determine whether or not they have been made (an alternative interpretation is that β and σ represent unverifiable effort decisions). As a result of these specific investments, the buyer and seller are to some extent locked into each other. In fact, to simplify matters, we suppose that the lock-in is complete, in the sense that by the time date 1 arrives, neither the buyer nor the seller can trade with any other party. At date 0, in contrast, we suppose that there are many similar parties with whom the buyer and seller can form a relationship, and that the division of ex-ante surplus between the two of them is determined in a competitive market for contracts. Let $\overline{U}$ be the market equilibrium expected profit level for the seller.

[4] Risk neutrality embraces the assumption that neither of them faces the possibility of bankruptcy.

The realizations of v and c are determined by β, σ, and the state of the world at date 1, ω; i.e. we have

$$(2.1)\qquad \begin{aligned} v &= v(\omega;\beta),\\ c &= c(\omega;\sigma), \end{aligned}$$

where $\omega \in \Omega$, the set of all states of the world, $\beta, \sigma \in \Gamma$, and v and c are functions mapping $(\Omega;\Gamma) \to R$. Equation (2.1) embodies the assumption that each party's investment affects only his own payoff, i.e., there are no externalities. To simplify, we suppose that Ω is finite. The state ω is assumed to be determined at date 1, and to be publicly observable but it is supposed to be sufficiently complex that state-contingent contracts cannot be written at date 0. The realizations of v and c are assumed *not* to be publicly observable (so that contracts contingent on them cannot be written), although we shall suppose that they are observable to the buyer and seller.[5,6] Finally, the joint distribution of v and c, as a function of β and σ, is assumed to be common knowledge at date 0.

The period between date 1 and date 2 can be used by the parties to revise and/or renegotiate the initial contract.[7] The sequence of events is illustrated in Figure 1.

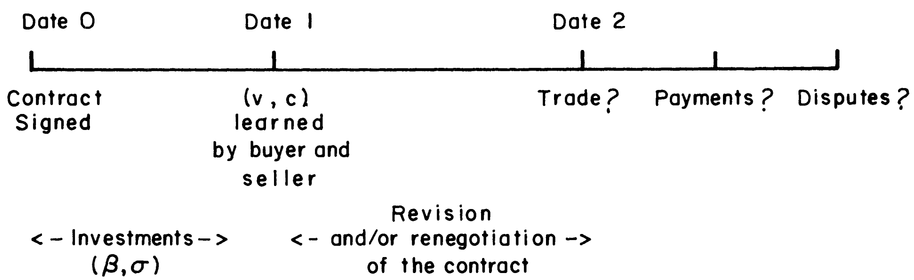

FIGURE 1

The physical mechanism by which, at date 2, trade is (or is not) consummated is important to the analysis. We take this mechanism as being exogenously given, and, specifically, we shall assume that trade occurs if and only if the seller is willing to supply the good and the buyer is willing to take delivery of it; in stylized terms one can imagine that the buyer and seller each has a switch that can be on or off, with trade occurring only if both switches are on. In the next section, in Remark (a) following Proposition 1, we discuss some consequences of making different assumptions about the technology of trade.

[5] The symmetry of information between the buyer and the seller can be justified on the grounds that they have a close relationship with each other.

[6] In order to justify the assumption that v, c are not observable to a court, we must suppose that these are private benefits and costs which accrue directly to the managers of the two firms—i.e., like effort, they don't show up in the accounts. More generally, actual benefits and costs might be observable, but the relationship between these observable variables and the unobservable effort levels of managers might be uncertain. It seems likely that our analysis could be extended to this case.

[7] We suppose that this revision and/or renegotiation is costless, clearly an extreme assumption.

We assume that if the good is traded after date 2, its value drops to zero (and it is not ready to be traded before date 2). Typically, some payment will be made by the buyer to the seller and this is supposed to occur shortly after date 2. Any disputes between the buyer and seller over whether the contract has been carried out also occur after date 2 and are resolved by a court (in fact, as we shall see, in equilibrium, no disputes actually occur).

It is important to emphasize that at all three dates 0, 1, and 2, the buyer and seller have symmetric information. As we shall see, the difficulty that the parties face is conveying this common information to the court. That is, in modern parlance, contractual problems arise because the parties' information, although observable to the parties, is not *verifiable* to outsiders.

With this background, let us now turn to the form of an optimal contract at date 0.[8] Let $q = 0$ or 1 be the amount traded between the buyer and seller at date 2. Clearly for trade to be efficient, we must have

$$(2.2) \qquad q = 1 \Leftrightarrow v \geq c.^{9}$$

The expected total surplus, given investments β and σ, is then

$$(2.3) \qquad W(\beta, \sigma) = E_{v,c}[\max\{v - c, 0\} | \beta, \sigma] - h_b(\beta) - h_s(\sigma)$$

where $h_b(\beta)$ and $h_s(\sigma)$ are the buyer and seller's respective costs of investment, and where the expectation operator, $E_{v,c}[\cdot | \beta, \sigma]$ is over the joint distribution of (v, c) defined by (2.1). (All benefits and costs are measured in date 2 money.) Let β^* and σ^* be the (assumed unique) investment levels which maximize expected total surplus $W(\beta, \sigma)$. Were it possible to contract over investment levels, an optimum contract would stipulate that between dates 0 and 1 the buyer and seller must undertake β^* and σ^* respectively; if a party failed to invest at the specified level, then he would be made to pay a large penalty to the other. (Given risk neutrality, the precise distribution of date 2 surplus, $\max\{v - c, 0\}$, would be unimportant; it would only matter that the seller had expected utility $\bar{U}$ overall, and if necessary this could be achieved by means of a side-payment at date 0.) However, since it is not possible to contract over investment levels, these have to be implemented *indirectly*, by dividing the date 2 surplus so as to give the parties appropriate private incentives at date 0.

A point to note in passing is that the trading rule (2.2) can be implemented by the parties *without* a contract at date 0; they can simply wait until date 1 and bargain (efficiently) then about how the ex post surplus should be divided. (A sidepayment will generally be needed at date 0, though, in order that the seller has utility $\bar{U}$.) The problem with this arrangement, of course, is that the division of surplus arising out of simple bargaining at date 1 will typically provide the buyer and/or seller with poor incentives to invest between dates 0 and 1 (see Grout (1984)). All this amounts to reiterating the basic thesis that long-term

[8] We confine attention to a contract between a separately owned buyer and seller; that is, we do not consider the possibility that the parties might resolve their contractual problems by vertically integrating. For an analysis of this, see Grossman and Hart (1986).

[9] We adopt the convention that $q = 1$ if $v = c$.

contracts are useful, if not essential, in facilitating trade where specific investments are involved (see Williamson (1985) and Klein, Crawford, and Alchian (1978)).

We have argued that the courts cannot observe (v, c).[10] What can they observe? We suppose that in the event of a dispute, all they can determine is (i) whether trade occurred or not, i.e. whether $q = 0$ or 1; (ii) how much the buyer paid the seller; (iii) certain messages, letters, or written documents that were exchanged by the buyer and seller between date 1 and date 2. If we ignore (iii) for a moment, the implication of (i) and (ii) is that a contract between the buyer and seller can specify only two prices, p_0 and p_1, where p_i is the price the buyer must pay the seller if $q = i$ $(i = 0, 1)$. The effect of (iii) is to allow p_i to depend on some messages $\boldsymbol{m}$ exchanged by the buyer and seller between dates 1 and 2. Hence the contract can specify price *functions* $p_0(\boldsymbol{m})$, $p_1(\boldsymbol{m})$ rather than just numbers p_0, p_1.[11]

Implicit in (i) is the assumption that the courts cannot determine *why*, if $q = 0$, trade does not occur; that is, the courts cannot distinguish between nontrade due to the seller being unwilling to supply and nontrade due to the buyer being unwilling to take delivery (with reference to our switch analysis, the courts cannot determine whose switch is in the off position). This is an important simplification. For example, if the court can determine whether it is the buyer, the seller, or both, who is unwilling to trade, up to three different no trade prices can be enforced instead of one (see Remark (a) following Proposition 1).

Given the initial contract and an exchange of messages, it is easy to determine the conditions under which trade will take place. The buyer incurs no penalties (over and above having to pay $p_0(\boldsymbol{m})$ to the seller) from not accepting delivery, and neither does the seller from not supplying. Therefore, trade will occur if and only if

$$v - p_1(\boldsymbol{m}) \geq -p_0(\boldsymbol{m}) \quad \text{and} \tag{2.4}$$
$$p_1(\boldsymbol{m}) - c \geq p_0(\boldsymbol{m}).$$

The first part of (2.4) says that the buyer is better off with $q = 1$ than $q = 0$, and

[10] If the courts can observe v and c directly, then it may be possible to achieve the first-best. In particular, if v and c are statistically independent (given β, σ), it is not difficult to show that the first-best investments β^* and σ^* can be implemented by writing the trading rule (2.2) into the contract, and specifying that for each realization of (v, c)—trade or no trade—the buyer pays the seller

$$p(v, c) = \int_{\tilde{v} \geq c} \tilde{v}\, dF(\tilde{v}; \beta^*) + \int_{\tilde{c} \leq v} \tilde{c}\, dF(\tilde{c}; \sigma^*) + \text{constant}$$

where $F(\sigma; \beta)$ and $F(c; \sigma)$ are the respective distribution functions of v and c. Note that there are generally also other schemes which implement first-best—we do not have uniqueness here.

[11] $(p_0(\boldsymbol{m}), p_1(\boldsymbol{m}))$ can be thought of as a nonlinear price schedule. Alternatively, $p_0(\boldsymbol{m})$ can be thought of as the damages the buyer pays the seller (which might be negative) if "breach" occurs (in legal terms, these are "liquidated" damages).

the second part says that the same is true of the seller.[12,13] (2.4) can be written more compactly as

$$(2.5) \qquad q=1 \Leftrightarrow v \geq p_1(\boldsymbol{m})-p_0(\boldsymbol{m}) \geq c.^{14}$$

This same "voluntary" trading rule, but without the messages, is considered in Hall and Lazear (1984) and Grossman and Hart (1987).

Before proceeding, we must mention an important implicit assumption that we have made. This is that it is impossible for the buyer and seller to include a *third party* in the date 0 contract, with this third party acting as a financial wedge between the buyer and seller. In this case the price paid by the buyer in a particular state does not have to equal the price received by the seller, with the third party making up the difference. In footnote 20 below, we give some reasons why three party contracts of this sort may be difficult to implement.

Returning to the two party situation, let us consider now the exchange of messages between dates 1 and 2. To fix ideas, imagine that the date 0 contract specifies only two prices p_0, p_1 rather than two price functions. If $v \geq p_1 - p_0 \geq c$ at date 2, trade will take place at these prices. However, suppose that $v > c$ but either $p_1 - p_0 > v$ or $c > p_1 - p_0$. Then even though there are gains from trade, they will not be realized under the contract. Hence the exchange of messages can be seen as a way of *revising* the contract.

In order to analyze the exact way in which revisions occur, it is necessary to be very precise about the message technology (the reader may be puzzled or even amused by the amount of detail that follows, but the rationale for it will, we hope, shortly become clear). We imagine that the time between dates 1 and 2 is divided up into a number of "days." Messages or "letters" are sent by a totally reliable "mail" service and take a day to arrive. There is one collection and one delivery of mail a day (for both the buyer and seller). Delivery of day $(i-1)$'s mail occurs before collection of day i's mail. The buyer and seller can send each other messages on the same day (i.e. simultaneously), and can send several messages at the same time.

It is supposed that messages can be sent on days 1 to d, and that a message sent on the last day, day d, arrives before the seller and buyer decide whether to trade at date 2.

We assume throughout:

(*) Messages cannot be forged. That is, the buyer, say, cannot claim that the seller sent him a message when in fact he really did not. It can be imagined

[12]Throughout, we shall ignore any equilibrium in which, at some date 2 node, one party is unwilling to trade simply because the other party is unwilling to trade, and vice versa; such an equilibrium would not be trembling-hand perfect.

[13]We suppose that c is a variable cost which is incurred only if trade occurs.

[14]If the courts can observe v and c directly, then the efficient trading rule (2.2) can always be implemented by designing the date 0 contract so that, whenever $v \geq c$, the difference between the trade price, $p_1(v, c)$, say, and the no-trade price, $p_0(v, c)$, lies in the interval $[c, v]$. This form of contract would be required in footnote 10 to obtain the first-best (i.e., to obtain efficient investments β^* and σ^* together with efficient trading at date 2).

that all messages are signed by the person who sent them and signatures cannot be forged.

Although messages cannot be forged, it does not follow from this that the recipient of a message cannot deny that he received it. In fact, we shall distinguish between two cases:

(A) It is impossible to record publicly a message sent by one party to another. If one party receives a message, he can choose to reveal it in the event of a dispute, but is under no obligation to do so since he can always deny that he received it.

(B) It is possible to record publicly a message. Hence a party cannot deny receipt of such a message.

Case (A) corresponds to the usual mail service. Case (B) corresponds, say, to the case where messages can be transmitted by telephone and the conversations can be recorded (and the recordings cannot be meddled with); or to the case where there is a reliable witness to a communication (an example of such a witness might be a telegraph company if the messages are telegrams).[15]

It turns out that the form of the optimal contract is very sensitive to whether (A) or (B) applies. In the next section, therefore, we analyze Case (A) and in the following section Case (B). In both sections we make one further assumption:

(**) There is nothing to stop the two parties agreeing at any time to tear up, or rescind, the date 0 contract and write a new one.

Assumption (**) seems very reasonable. First, it corresponds to the way contracts are treated under the law. Secondly, it is hard to see how the parties could constrain themselves in advance not to revise a contract (but see footnote 19).[16]

3. CASE (A): SENDING A MESSAGE CANNOT BE VERIFIED

The task facing the buyer and seller at date 0 is to design a revision, or message, game to be played from date 1. Since it is too costly to contract over the

[15] The witness is a third party, and hence the same factors which make the inclusion of any third party in the contract problematical may be relevant here (see footnote 20). Note that registered mail does not satisfy the conditions of Case (B) since, although this allows one to establish that a message was sent, there is no record of its contents. In principle, a computer mail system could be designed to keep not only a record of the contents of a message, but also a (verifiable) record of the fact that the message was sent; this would fall into Case (B).

[16] It should noted that we do not allow the parties to send messages after they have made their investment decisions, but before v, c are realized. Two justifications can be given for this. First, such messages become useful only when each party learns the other party's investment decision, and this may not happen until (close to) date 1. Secondly, the information about the other party's investment decision may never become available, i.e. β, σ may remain private information despite the fact that v, c *are* observed by the two parties.

investments β and σ directly, expected total surplus has to be maximized by instead dividing the date 2 surplus in such a way as to give each party the right private incentives to invest. An optimal revision game will yield trade and price outcomes that are appropriately sensitive to the realized pair (v, c).

In principle, this game can be very complex: it may involve many moves by each player, some of which are simultaneous, some of which are sequential. A natural approach is to study the subgame perfect equilibria of each possible game and, in light of this, to consider what is the optimal revision game for the buyer and seller to select at date 0. At first sight this exercise seems daunting. However, as we shall show in this section and the next, it turns out that, by virtue of (**), the possibilities at the disposal of the two parties are actually quite limited. And if they cannot arrange for messages to be verified (Case A), then the limitations are severe.

The revision game really consists of two subgames, one of which is the pure message game played between date 1 and date 2, and the other the dispute game played after date 2. In the dispute game, each party decides which of the messages received from the other party to reveal to the court (a decision which may depend on the strategy which the party expects the other party to follow in this regard). This dispute game is played *after* q has been chosen, its concern being the price that should be paid—p_1 if q was equal to 1, and p_0 if q was equal to 0. Note that both subgames are games of complete information, since v, c are known to both parties at date 1.[17]

We observe first that in Case (A), neither party can be forced to send a message; that is, the date 0 contract cannot, for example, penalize the buyer for not sending a message by raising p_0 and p_1, since the seller can (and will) then increase his profit by denying that the message was received. This is one way in which a third party could be helpful: the buyer's penalty for not sending a message could be paid to the third party rather than to the seller, which would remove the seller's incentive to deny receipt. In Section 2, however, we ruled out third parties, an assumption which is partially justified in footnote 20.

Given that the parties cannot be forced to send messages, the contract must specify the prices $\hat{p}_0$, $\hat{p}_1$, say, which will apply if no messages are sent. One can think of $\hat{p}_0$, $\hat{p}_1$ as being "default" or "status quo" contract prices, i.e. prices that rule in the absence of revisions. We now show that, in Case (A), for each realization of v and c, the revision game has only *one* possible equilibrium outcome—which is determined entirely by the values of $\hat{p}_0$ and $\hat{p}_1$. The way in which an equilibrium trading rule and prices depend on $\hat{p}_0$ and $\hat{p}_1$ is described in Proposition 1.

PROPOSITION 1: *Let* $(\hat{p}_0, \hat{p}_1)$ *be the prices which the date* 0 *contract specifies will apply if no messages are sent between dates* 1 *and* 2. *Then, in Case* (*A*), *the*

[17] This is clear if they have observed each other's investments β and σ. However, our results would be unchanged if β and σ remained private information, because $h_b(\beta)$ and $h_s(\sigma)$ are sunk costs and only v and c are payoff-relevant in the revision game.

trading rule and price which will obtain at date 2 are as follows:

(1) *If* $v < c$, $q = 0$ *and the buyer pays the seller* $\hat{p}_0$.

(2) *If* $v \geq \hat{p}_1 - \hat{p}_0 \geq c$, $q = 1$ *and the buyer pays the seller* $\hat{p}_1$.

(3) *If* $v \geq c > \hat{p}_1 - \hat{p}_0$, $q = 1$ *and the buyer pays the seller* $\hat{p}_0 + c$.

(4) *If* $\hat{p}_1 - \hat{p}_0 > v \geq c$, $q = 1$ *and the buyer pays the seller* $\hat{p}_0 + v$.

Proposition 1 is illustrated in Figure 2, where $k \equiv \hat{p}_1 - \hat{p}_0$, and v varies between $\underline{v}$ and $\bar{v}$, and c between $\underline{c}$ and $\bar{c}$.

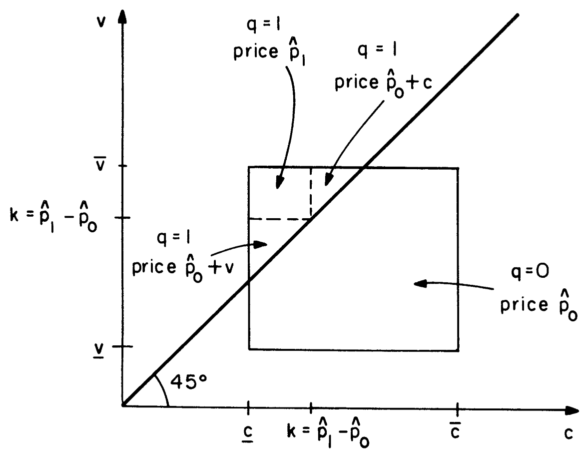

FIGURE 2

A formal proof of Proposition 1 is given in Appendix A. There, we deal with certain technical points which strictly ought to be resolved before the revision process can be analyzed as a formal game. In particular, we have to consider how the dispute subgame is played after date 2, and how the courts would rule if faced with conflicting evidence. It turns out that Proposition 1 remains true no matter how these points are dealt with, and since some of the considerations are arcane, we have relegated them to the Appendix.

The intuition behind Proposition 1 is as follows. Consider first the region

(3.1) $v < c$.

Whatever messages are sent and revealed, the parties know that (2.5) cannot be satisfied. Hence they know that trade will not occur at date 2, and the relevant price is $p_0(\boldsymbol{m})$. This means that the buyer and seller are playing a zero-sum game over this price. Given that each can hold the other to $\hat{p}_0$ by sending no messages, and revealing none from the other party in the event of a dispute, it follows that the unique equilibrium price is $\hat{p}_0$. That is, over the region $v < c$, $p_0 \equiv \hat{p}_0$ is independent of (v, c).

A similar argument applies to the dotted box region,

(3.2) $v \geq \hat{p}_1 - \hat{p}_0 \geq c$.

Suppose first the buyer and seller send no messages. Then when date 2 arrives, the seller will wish to supply and the buyer will wish to accept the good, and so trade will occur at the price $\hat{p}_1$. That is, it is feasible for trade to occur without any revisions being made to the contract. Might trade take place at any other price? The answer is no, since, if the price were higher as a result of certain messages or revisions, the buyer could do better by sending no messages and revealing none from the seller, which would guarantee him a price of $\hat{p}_1$. Similarly, if the price were lower than $\hat{p}_1$, the seller could do better by sending no messages and revealing none from the buyer. That is, each party can hold the other to trade at the price $\hat{p}_1$, and so this must be the unique equilibrium outcome.

Although this argument is very similar to that given for the no-trade region $v < c$, it is in fact a bit more subtle. The game is not zero sum anymore since even though trade can take place at the unrevised contract prices $\hat{p}_0$, $\hat{p}_1$, the parties may exchange messages in such a way that the final prices $p_0(\boldsymbol{m})$, $p_1(\boldsymbol{m})$ do not satisfy (3.2), in which case trade will not occur. In view of this, one party may try to threaten the other. For example, the buyer may send a letter to the seller saying, "If you don't agree to a price substantially below $\hat{p}_1$, I won't buy from you." Such a threat is not credible, however, since the seller knows that if he ignores the letter and, come date 2, supplies the good, it will be accepted. So as long as we include the requirement of perfection or credibility, $\hat{p}_1$ is the unique equilibrium price in the dotted box.

Consider next the triangle North-East of the dotted box, where

$$(3.3) \qquad v \geq c > \hat{p}_1 - \hat{p}_0.$$

The seller can always guarantee himself a net return of $\hat{p}_0$ by sending no messages to the buyer, refusing to deliver at date 2, and denying receipt of any messages in the event of a dispute. Hence if trade does occur, the buyer must pay the seller *at least* $\hat{p}_0 + c$. Proposition 1 contains the striking conclusion that the buyer need pay no more. The way the buyer achieves this is as follows. First, he sends no messages to the seller until the last mail day before date 2. Then on this last mail day, he sends the following letter: "I propose that we rescind the old contract, and write a new contract with prices ($\hat{p}_0$, $\hat{p}_0 + c$). If you agree, sign here and retain." Note that the buyer is offering to raise the price in this new contract —since $\hat{p}_0 + c > \hat{p}_1$ by (3.3).

This proposed new contract has the following effect. The seller will now be prepared to supply the good at date 2 since he knows that, if there is a dispute, he can always sign the new contract and produce it as evidence to obtain the price $\hat{p}_0 + c$. Note that at the unrevised prices ($\hat{p}_0$, $\hat{p}_1$), the seller would *not* agree to supply the good since his net return $\hat{p}_1 - c$ is below $\hat{p}_0$.

The question arises: can the seller obtain a trading price *above* $\hat{p}_0 + c$? The answer is no. For example, consider the following letter that the seller might send the buyer two days before date 2: "I propose we rescind the old contract, and write a new contract with prices ($\hat{p}_0$, $\hat{p}_0 + v$). Please write back tomorrow confirming that you agree. If you do not, I will not supply." The seller is attempting to raise the trading price (up to the point at which the buyer would only just be willing to trade). Notice that the seller must allow the buyer time to reply before date 2 (otherwise, in the event of a dispute, the buyer can always claim that the original contract applies)—which is why the seller has to send his letter two days earlier. Unfortunately this attempt fails, because the buyer can simply ignore the seller's letter, and, on the day before date 2, send his own letter offering prices $(\hat{p}_0, \hat{p}_0 + c)$. As we have explained, the seller will be willing to supply the good (despite his threat not to) because he can always produce the buyer's letter after date 2, with his own signature added, to obtain at least price $\hat{p}_0 + c$. Meantime the buyer will simply conceal the seller's letter from the courts in the event of a dispute. (Actually, even if he produces it, it would not have any effect as long as he hasn't agreed to it by signing it.)

At first sight it may seem odd that the buyer is able to get all the gains from trade under (3.3) (i.e. the seller is indifferent between not trading under the old contract and trading under the new one). The reason for this apparent asymmetry is that when (3.3) holds, the buyer prefers to trade than not at the unrevised prices $(\hat{p}_0, \hat{p}_1)$, while the seller does not. So it is the seller who wants, and hence will sign, a new contract, and this gives the buyer the power to dictate terms in what amounts to a "take it or leave it" contract which he offers on the last day before date 2.

In the triangle South-West of the dotted box, where

$$(3.4) \qquad \hat{p}_1 - \hat{p}_0 > v \geq c,$$

the asymmetry works the other way. Now the seller has all the power. On the last day before date 2, he offers the new contract $(\hat{p}_0, \hat{p}_0 + v)$ and the buyer is then prepared to accept the good, knowing that he will only have to pay $\hat{p}_0 + v < \hat{p}_1$. The seller can dictate terms here because he is happy to trade under the old contract, while the buyer wants a new contract.

This completes our informal discussion of Proposition 1. In actual fact, the prices and quantities specified in Proposition 1 can be implemented without a new contract having to be written at date 1. A simple date 0 contract which achieves this is:

> In the event of trade, the payment (from buyer to seller) will be $\hat{p}_1$—unless either the buyer offers to make a higher payment or the seller offers to accept a lower one. In the event of no trade, the payment will be $\hat{p}_0$.

Two special cases of Proposition 1 are worth noting. The first is where $\hat{p}_1 - \hat{p}_0 \leq \underline{c}$ in Figure 2. Then the trading price is $\hat{p}_0 + c$, and the buyer has all the power over the whole region $v \geq c$. The second is where $\hat{p}_1 - \hat{p}_0 \geq \bar{v}$, in which case the trading price is $\hat{p}_0 + v$, and the seller has all the power over the whole region $v \geq c$.

The results of this section can be summarized as follows. When messages cannot be verified, the ability of the buyer and seller to limit the way contractual revisions are made in the future is very small. In particular, once $\hat{p}_0$ and $\hat{p}_1$ are specified, prices in all states (v, c) are determined according to Figure 2. In fact, the parties have only *one* degree of freedom rather than two, since given $k \equiv \hat{p}_1 - \hat{p}_0$, $\hat{p}_0$ and $\hat{p}_1$ will adjust so that the seller's expected profit equals $\overline{U}$, determined in the ex ante market for contracts. In the next section, we shall see that the parties have many more degrees of freedom if messages between dates 1 and 2 can be verified (Case (B)).

Before going on to Section 4, it is important to know to what extent Proposition 1 is sensitive to the simplifying assumptions we have made. Unfortunately, given limitations on space, we cannot do justice to the many possible extensions, but merely confine ourselves to two.

(*a*) *The Trading Mechanism*

Our assumption has been that the courts cannot determine why trade did not occur. Suppose instead that they can. We now briefly consider two possible trading mechanisms: (i) the buyer and seller simultaneously have to decide whether to trade; and (ii) the seller first decides whether to supply the good and then the buyer decides whether to take delivery. In both cases it is possible to implement a richer set of final prices than those in Proposition 1. (Note that if the courts could not determine why trade did not occur, then it would not matter which of (i) or (ii) applied.)

We start with trading mechanism (i). Up to three different no-trade prices can be usefully specified in the date 0 contract: $\hat{p}_0^B$, the price if only the buyer refuses to trade; $\hat{p}_0^S$, the price if only the seller refuses to trade; and $\hat{p}_0^{BS}$, the price if both parties refuse to trade. Now *without* renegotiation there may be multiple equilibria at date 2. For example, if $\hat{p}_0^B = 3$, $\hat{p}_0^S = 1$, $\hat{p}_0^{BS} = 2$, and $\hat{p}_1 = 9$, then if $(v, c) = (5, 10)$ there are two pure-strategy equilibria—one in which the buyer refuses to trade, and another in which the seller refuses to trade. However, we can show that in all cases, the renegotiation game leads to efficient trading, and a unique set of prices. In fact, in terms of Figure 2, the $v - c$ space can be divided into a maximum of ten, rather than just four, distinct regions. And not surprisingly, there is some variability in price across the no-trade region, $v < c$—unlike in Proposition 1.

Turn now to trading mechanism (ii). Two distinct no-trade prices can be usefully specified in the contract: $\hat{p}_0$ if the seller fails to supply, and $\hat{p}_0'$ if the buyer refuses to take delivery. Now without renegotiation there is no problem with multiple equilibria at date 2, since the trading game has two stages: the seller moves before the buyer. However, the renegotiation game may itself have multiple equilibria. For example, if $\hat{p}_0 = 100$, $\hat{p}_0' = 89$, and $\hat{p}_1 = 120$, then if $(v, c) = (30, 10)$ there are two pure-strategy equilibria in the message game:

> On the last day before date 2 the buyer sends the message: "I propose we raise p_0' to 100." This results in trade at price 120.

On the last day before date 2 the seller sends the message: "I propose we lower p_1 to 119." This results in trade at price 119.

Our general conclusion is that the results are sensitive to the trading mechanism and, particularly, to what the courts can retrospectively determine. Nevertheless, the analysis of Proposition 1 is still very suggestive of which trading prices can be implemented under various assumptions.

(*b*) *The Message Technology*

It seems natural to assume that in principle messages can be arbitrary, multiple, and sent simultaneously. But nothing hinges on this assumption. Suppose that messages can only be sent singly, and that the buyer and seller have to alternate. Take the case where the seller is unwilling to trade under the old prices $(\hat{p}_0, \hat{p}_1)$; for example let the old contract be $(\hat{p}_0, \hat{p}_1) = (0,0)$—or, equivalently, there is no old contract—and let $v = 5$ and $c = 3$. The buyer can still obtain all the surplus by offering a new contract containing prices $(0,3)$ on the last day before date 2 that he is permitted to send a message—regardless of whether or not the seller has time to reply. The point is that the seller cannot usefully reply by rejecting these terms and offering $(0,5)$ instead, because there would be no time left for the buyer to say "yes" before date 2. (And if the buyer did have an opportunity to send a further message, it would of course not be "yes," but a repeat of his earlier $(0,3)$ offer!) All that matters is that each party has an opportunity to send at least one message.

Comment (b) suggests a comparison with the usual finite-horizon noncooperative bargaining model (based on the infinite-horizon model of Rubinstein (1982)), where inter alia, simultaneous offers are ruled out. There, in contrast to the above, it matters a great deal who can make the final offer. For the example $(v, c) = (5,3)$, there will be trade at price 3 or 5 depending on whether it is the buyer who offers last or the seller. This contrast with our model stems from three differences between the models. (1) In our model trade can occur irrespective of whether agreement has been reached on price. To put it formally, even in the restricted case of alternating offers with the seller on day d making the last offer, we include a "final" stage in the game, at date 2, when trade can occur. (In fact this may not be the final stage, if there is a dispute.) (2) We draw no distinction between types of messages. In the usual bargaining model, there is a qualitative difference between making an offer and accepting/rejecting one: when the seller is "the last to make an offer," this really means that the buyer subsequently has time to accept or reject this offer but cannot send a more complicated message in reply (e.g., a counter-offer). (3) Offers (letters) in our model are durable. Again to use the example, the seller can keep the offer of $(0,3)$ sent by the buyer on day $d-1$ and produce it before the court in the event of a dispute after date 2. By contrast, in the usual bargaining model (verbal?) offers "die" once they have been made and rejected.

We should stress, though, that the equilibrium in the usual bargaining model (viz., trade at price 3 or 5) would not be affected by having offers that are durable. Nor would it matter much if no distinction were drawn between types of message (except that all the surplus would go to the agent who *didn't* have the opportunity to send the last message, because in equilibrium this last message will simply be "accept"). It is the *combination* in our model of being able to trade without agreement on price, and having durable (and general) messages, which leads to trade at price 3.

4. CASE (B): SENDING A MESSAGE CAN BE VERIFIED

There are clearly many situations in which it is feasible for an agent to send a message which is publicly verifiable. It is therefore important to extend the analysis of the previous section to cover such cases. We shall see that the result in Proposition 1 still has considerable relevance here, since an agent could (if he chose to) send a message 'privately'—that is, in a way that cannot be publicly verified.

A major difference between Cases (A) and (B) is that, when messages can be verified, the date 0 contract can force each party to send one or more messages from a prescribed set. That is, suppose it is ex-ante desirable for the buyer to send one of the messages $b_1, b_2, \ldots, b_m$ and the seller to send one of the messages $s_1, s_2, \ldots, s_n$ between dates 1 and 2. Then, in Case (B), this can be ensured by a provision which says that the buyer (resp. seller) must pay the seller (resp. buyer) a large sum if he sends a message other than exactly one of $b_1, \ldots, b_m$ (resp. $s_1, \ldots, s_n$) or doesn't send any message at all.

For reasons which will become clear shortly, it is convenient to consider the message game in normal form. As above, let the messages—or strategies—of the buyer and seller be $b_1, \ldots, b_m$ and $s_1, \ldots, s_n$, respectively. Any pair of messages (b_i, s_j) leads to "revised" contract prices, denoted by (p_0^{ij}, p_1^{ij}). That is, if the buyer sends the message b_i and the seller sends the message s_j, the resulting price will be p_1^{ij} if they trade at date 2 and p_0^{ij} if they don't. The messages $b_1, \ldots, b_m$ and $s_1, \ldots, s_n$, and the mapping from messages to prices given by the pairs (p_0^{ij}, p_1^{ij}), are choice variables in the date 0 contract.

Although (p_0^{ij}, p_1^{ij}) are revised contract prices, they may not be final prices. The reason is the following. Suppose $v \geq c$, the buyer sends b_i and the seller sends s_j, but $p_1^{ij} - p_0^{ij}$ does not lie between v and c. Then although trade is mutually beneficial, it will not take place under the revised contract. However, it will then be in the interest of the two parties to rescind the revised contract and write a new contract which enables trade to occur. We suppose that this happens in the same way as in Section 3. That is, if $v \geq c$ the final trading price will be:

$$(4.1) \qquad p_1^{ij}(v, c) = \begin{cases} p_1^{ij} & \text{if} \quad v \geq p_1^{ij} - p_0^{ij} \geq c, \\ p_0^{ij} + c & \text{if} \quad v \geq c > p_1^{ij} - p_0^{ij}, \\ p_0^{ij} + v & \text{if} \quad p_1^{ij} - p_0^{ij} > v \geq c. \end{cases}$$

On the other hand, if $v < c$, trade will not occur and the price will be p_0^{ij}.

We see then that the possibility that the contract can be renegotiated has an important implication.[18] The date 0 contract cannot make the "revised" trading prices depend directly on v, c (since v, c are not publicly observable), but only indirectly via the messages b_i, s_j sent. However, it is clear from (4.1) that renegotiation can lead to a final trading price which depends directly on (v, c). Note that this is not true of the no trade price, p_0^{ij}, which rules if $v < c$, and which depends only on b_i, s_j.

Let us return to the game, given a particular pair v, c. Suppose first that $v \geq c$. Then it follows from the above argument that, *whatever* messages b_i, s_j are sent, trade will occur. This means that the buyer and seller are playing a *zero-sum game* where the payoff, $p_1^{ij}(v, c)$, defined in (4.1), is the amount the seller receives from the buyer (this payoff ignores the buyer's value v and the seller's cost c). Let $p_1^*(v, c)$ be the *value* of this game, defined by

$$(4.2)\quad \begin{gathered} p_1^*(v,c) = \min_{\pi} \max_{\rho} \sum_{i=1}^{m} \sum_{j=1}^{n} \pi_i \rho_j p_1^{ij}(v,c), \quad \text{where} \\ \pi \in \left\{ \pi \mid \pi \geq 0,\ \sum_{i=1}^{m} \pi_i = 1 \right\}, \qquad \rho \in \left\{ \rho \mid \rho \geq 0,\ \sum_{j=1}^{n} \rho_j = 1 \right\}. \end{gathered}$$

Then, by the well-known saddle point property (see, e.g., Luce and Raiffa (1958)), all Nash equilibria of this game (some of which may involve mixed strategies) give the seller an expected payoff of $p_1^*(v, c)$ and the buyer an expected payoff of $-p_1^*(v, c)$.

If $v < c$, the game is again zero-sum, where this time the payoff is p_0^{ij}. We denote the value of this game by

$$(4.3)\quad p_0^* = \min_{\pi} \max_{\rho} \sum_{i=1}^{m} \sum_{j=1}^{n} \pi_i \rho_j p_0^{ij},$$

where π, ρ have the same domains as above. p_0^* is the expected amount the buyer pays the seller in the event of no trade. Again, while there may be multiple equilibria, they are equivalent for both the buyer and seller.

The fact that the buyer and seller play a zero-sum game, both when $v \geq c$ and when $v < c$, explains our decision to analyze the game in normal form. Any subgame perfect equilibrium of the extensive form is one of the Nash equilibria of the normal form; but we now know that all Nash equilibria lead to a common expected price, given by (4.2) or (4.3) as $v \geq c$ or $v < c$ respectively.

The next proposition provides a complete characterization of the expected trade and nontrade prices which can be implemented in the case of verifiable messages.

[18] The exact way the contract is renegotiated may be more complicated than in Section 3. Suppose, for example, the messages b_i, s_j are sent on the last day, d. Then since there is no time left for renegotiation, the new contract must be exchanged at the same time. Given that it is not yet clear what the prices under the old contract will be or who has the power to dictate the terms of this new contract, one can imagine that each party sends a new contract on day d, proposing prices which are contingent on the message the other party sends on that day. By date 2, it will be clear what the old contract prices are and which of these new contracts has force; the revised prices will then be given by (4.1).

PROPOSITION 2: *Let* $p_1^*(v,c)$ *and* p_0^*, *defined in* (4.2) *and* (4.3), *be the values of the above game, respectively when* $v \geq c$ *and when* $v < c$. *Then*:

(1) *for all* $v \geq c$, $p_1^*(v,c)$ *is nondecreasing in* v *and* c;

(2) *if* $v' \geq c'$, $v \geq c$, *and* $p_1^*(v',c') > p_1^*(v,c)$, *then*:

$$p_1^*(v',c') - p_1^*(v,c) \leq \max\{v'-v, c'-c\};$$

(3) *for all* $v \geq c$, $p_1^*(v,c) - v \leq p_0^* \leq p_1^*(v,c) - c$.

Furthermore, given any price function $\tilde{p}_1(v,c)$ *and price* $\tilde{p}_0$ *satisfying* (1)–(3), *we can find a game which has these as the associated values.* (*If* v *is never less than* c (*so that* $\tilde{p}_0$ *is not defined*), *then condition* (3) *should be prefaced by "there exists a* $\tilde{p}_0$ *such that* . . . .")

Conditions (1)–(3) are not surprising. (1) says that the price the buyer must pay for the good cannot fall if the seller's cost rises or if the buyer's valuation rises. (3) says that neither the buyer nor the seller can be worse off trading than not. (2) is a bit less intuitive, but it says, among other things, that if v and c rise by α, p_1^* rises by no more than α. Note that the class of price functions satisfying (1)–(3) is much richer than that satisfying Proposition 1 (which applies to the case of nonverifiable messages).

PROOF OF NECESSITY:[19] To prove the necessity of (1), note that the final price contingent on messages b_i and s_j being sent, given by $p_1^{ij}(v,c)$ in (4.1), is nondecreasing in v and c. Condition (1) then follows directly from (4.2).

To prove the necessity of (2), set $\alpha = \max\{v'-v, c'-c\}$. If $p_1^*(v',c') > p_1^*(v,c)$, then from (1), $\alpha > 0$. If $v'-v = c'-c = \alpha$, it follows from (4.1) that

$$p_1^{ij}(v',c') - p_1^{ij}(v,c) \leq \alpha \qquad \text{for all } i \text{ and } j.$$

Hence, again using (1), we see that this last inequality holds if either $v'-v < c'-c = \alpha$ or $c'-c < v'-v = \alpha$. Condition (2) then follows directly from (4.2).

Finally, to prove the necessity of (3), observe that, from (4.1),

$$p_1^{ij}(v,c) - v \leq p_0^{ij} \leq p_1^{ij}(v,c) - c \qquad \text{for all } i \text{ and } j.$$

Condition (3) then follows directly from (4.2). *Q.E.D.*

[19]We are grateful to Eric Maskin for providing the argument for necessity. We should also note that the proof of sufficiency bears a resemblance to Maskin's work on the implementation of Nash equilibrium (see Maskin (1986)). The main difference is that we analyze a multi-stage game and allow for renegotiation.

It can be shown, both in the nonverifiable and verifiable cases, that the possibility of ex post renegotiation reduces the set of feasible contracts ex ante. In view of this, one might ask whether the parties can constrain themselves not to use the renegotiation option. One possibility is for them to agree that any suggestion by one to the other (through the mail, say) that the old contract should be rescinded should be heavily penalized. This may be difficult to arrange for three reasons. First, certain rescissions and negotiations may be desirable (although we have not modelled this), and it may be difficult to specify in advance which these are. Second, there may be a number of ways of modifying a contract which are less visible than tearing it up, and it may be difficult to find a general way of ruling these out. Third, the party proposing rescission could arrange to have the new contract taken personally to the other party, with the instruction that it should be released only once it has been signed by this other party; the new contract, moreover, could contain a clause waiving the penalty.

The (constructive) proof of the sufficiency of (1)–(3) may be found in Appendix B.

5. OPTIMAL CONTRACTS

We have characterized the ex-post division of surplus that the parties can achieve, both for the case where messages cannot be verified and for the case where they can be. We now consider what implications our results have for the form of an optimal second-best contract.

There are certain special cases in which it is possible to achieve the first-best: that is, where the ex post surplus can be divided in such a way so as to give the buyer and seller the correct private incentives to invest. These cases are grouped together in the next Proposition. Notice that in each case it is enough to use *nonverifiable* messages.

PROPOSITION 3: *The first-best can be achieved using a nonverifiable message scheme if any one of the following conditions holds*:

(1) *there exists some* k *for which* $v \geq k \geq c$ *with probability* 1 *for all* β, σ;

(2) $v(\omega; \beta)$ *is independent of* β;

(3) $c(\omega; \sigma)$ *is independent of* σ;

(4) $v(\omega; \beta)$ *and* $c(\omega; \sigma)$ *are independent of* ω.

In all four cases, it is easy to see why the first-best can be implemented. In case (1), if the difference between $\hat{p}_1$ and $\hat{p}_0$ is set equal to k, then trade will always occur at price $\hat{p}_1$. Since neither the buyer nor the seller can influence the terms of trade, their private investment decisions only affect their own benefits or costs. There is therefore no externality, and the first-best is achieved. In case (2), only the seller's action matters. One simple way of getting him to make the efficient investment, σ^*, is to ensure that he receives all the ex post surplus. This can be done by setting the difference between $\hat{p}_1$ and $\hat{p}_0$ to be larger than the buyer's highest possible valuation, $\max_\omega v(\omega)$, so that the buyer never wants to trade under the original contract. As we have seen in Section 3, when the contract is revised at date 1, the trading price will be $\hat{p}_0 + v$—which gives the seller all the surplus. Of course, this is tantamount to a more conventional contractual arrangements in which p_0 is fixed and the seller has the flexibility to send a (verifiable) message offering a price p_1. Case (3), where only the buyer's action β matters, works the other way round: now choose $\hat{p}_1 - \hat{p}_0$ to be less than the seller's lowest possible cost, $\min_\omega c(\omega)$, so that the revised trading price is $\hat{p}_0 + c$, and the buyer makes the efficient investment, β^*. Finally, if there is no uncertainty (case (4)), either (i) the buyer and seller will want and expect trade, in which case each will make their first-best investment; or (ii) they will not want or will not expect trade, in which case each will make their minimum investment. The first-best can be achieved by dividing the surplus in such a way as to make both parties better off than if there were no relationship: that way, (i) is assured.

Conditions (1)–(4) in Proposition 3 are very strong. In general, the first-best cannot be achieved, as we see in Proposition 4. This Proposition considers a case where both investments matter and there is a possibility of no trade. With particular assumptions about the stochastic functions $v(\omega; \beta)$ and $c(\omega; \sigma)$, we are able to characterize an optimal second-best contract.

PROPOSITION 4: *If for all* (β, σ) *the random variables* $v(\cdot\,; \beta)$ *and* $c(\cdot\,; \sigma)$ *are statistically independent, and if* β *and* σ *can be scaled so that they both lie in* $[0,1]$, *and if:*

(1) *for each* β *in* $(0,1)$, *the (nondegenerate) support of* $v(\cdot\,; \beta)$ *is*

$$\{\underline{v} = v_1 < \cdots < v_i < \cdots < v_I = \bar{v}\} \qquad (I \geq 2)$$

and the probability of v_i *is*

$$\pi_i(\beta) = \beta\pi_i^+ + (1-\beta)\pi_i^-$$

where $\boldsymbol{\pi}^+$ *and* $\boldsymbol{\pi}^-$ *are probability distributions over* $\{v_1, \ldots, v_I\}$ *and* π_i^+/π_i^- *is increasing in* i;

(2) *for each* σ *in* $(0,1)$, *the (nondegenerate) support of* $c(\cdot\,; \sigma)$ *is*

$$\{\bar{c} = c_1 > \cdots > c_j > \cdots > c_J = \underline{c}\} \qquad (J \geq 2)$$

and the probability of c_j *is*

$$\rho_j(\sigma) = \sigma\rho_j^+ + (1-\sigma)\rho_j^-$$

where $\boldsymbol{\rho}^+$ *and* $\boldsymbol{\rho}^-$ *are probability distributions over* $\{c_1, \ldots, c_J\}$ *and* ρ_j^+/ρ_j^- *is increasing in* j;

(3) $h_b(\cdot)$ *and* $h_s(\cdot)$ *are convex and increasing in* $[0,1]$, *with*

$$\lim_{\beta \to 0} h_b'(\beta) = \lim_{\sigma \to 0} h_s'(\sigma) = 0$$

and

$$\lim_{\beta \to 1} h_b'(\beta) = \lim_{\sigma \to 1} h_s'(\sigma) = \infty;$$

(4) $\underline{v} < \bar{c}$ *and* $\bar{v} > \underline{c}$;

then the first-best cannot be achieved. Moreover, even if messages are verifiable, the second-best can be achieved using a nonverifiable message scheme. Finally, the second-best actions β *and* σ *are both strictly less than their respective first-best levels* β^* *and* σ^*.

PROOF: See Appendix C.

REMARK: The important conditions in Proposition 4 are (1) and (2). According to (1), the distribution of the buyer's valuation v is a convex combination of two probability vectors, $\boldsymbol{\pi}^+$ and $\boldsymbol{\pi}^-$. The vector $\boldsymbol{\pi}^+$ (first-order) stochastically dominates $\boldsymbol{\pi}^-$. (In fact, the vectors satisfy the (strict) Monotone Likelihood Ratio Condition.) A greater investment β puts more relative weight on the vector $\boldsymbol{\pi}^+$.

Condition (2) has a similar interpretation. These conditions amount to a combination of the Spanning Condition and the (strict) Monotone Likelihood Ratio Condition discussed in Grossman and Hart (1983, pp. 23 and 25). Condition (3) is unimportant; it simply ensures a unique, interior solution for β and σ. Finally if, contrary to Condition (4), $\underline{v} \geq \bar{c}$, then we know from Proposition 3(1) that it would be possible to achieve the first-best.

The reason the first-best cannot be achieved is that there is an externality, the nature of which is as follows. Consider the buyer's choice of investment, β. If he reduces β then he typically reduces his expected valuation (this is certainly the case in Proposition 4). But if, in some state, his valuation falls below the difference between the trading price and the no-trade price, then one of two things will happen. Either trade still takes place but at a lower price, or there is no trade because it is not efficient. In either circumstance the seller (as well as the buyer) loses out. So the buyer's action affects the seller. Equally, the seller's choice of investment can change the buyer's expected surplus.[20]

[20] Throughout we have ruled out a third party who acts as a financial wedge between the buyer (B) and seller (S). The inclusion of a (risk neutral) third party (T) makes it possible to achieve the first-best, using the following "Groves-type" mechanism. The contract states that (i) at date 1, B sends T a message announcing his benefit v_a, and S sends a message announcing his cost c_a; (ii) $q = 1$ if and only if $v_a \geq c_a$; (iii) if $q = 1$, B pays T c_a and T says S v_a, while if $q = 0$, payments are zero. This mechanism elicits the truth from B and S at date 1 since neither's payment depends on his own announcement. It also ensures efficient actions since B and S's payoffs (gross of effort) are both equal to social surplus, $\max\{(v - c), 0\}$. T makes an expected loss from participating in the contract, but he can be compensated by an appropriate sidepayment at date 0.

While there may be large potential efficiency gains from the inclusion of a third party, various practical problems may prevent these gains from actually being realized. The most serious of these involves the possibility of collusion by two of the agents against the third. (This point has been noted by Eswaran and Kotwal (1984).) For example, in the case described above, there is an incentive for B and T to write a new "side-contract" just after the initial three party contract is signed. This new side-contract says that all payments made by T to S under the original contract must be matched by payments from B to T and that all payments made by B to T must be returned. This arrangement is equivalent to a merger between B and T, with T's net payment becoming zero in every state, i.e. B buys T out. B's new payment, on the other hand, becomes $(c_a - c_a + v_a) = v_a$. Obviously T is indifferent to this merger. B cannot be worse off since he can always choose the same action as without the merger and, given that he is risk neutral, the change in the distribution of returns is of no consequence to him. (We are implicitly assuming that S doesn't observe the writing of the new contract until after he takes his action; otherwise *his* action might change.) In fact it is easy to show that B will be better off. Exactly the same argument shows that there is an incentive for S and T to merge.

One way to avoid these mergers, of course, is to prohibit them in the original contract. This may be problematical, however, for two reasons. First, there may be a perfectly legitimate reason for B and T (or S and T) to write certain sorts of new contracts with each other, and it may be difficult to specify in advance which new contracts are allowable and which are not. Secondly, the side-contract may be very complicated, involving subsidiaries of the two companies or intermediaries. For example, B might merge with X who might merge with Y... who might merge with T. It may be very difficult to give an exhaustive list in the date 0 contract of all illegitimate combinations of such side-arrangements.

If for these reasons, side-contracts cannot be prevented, the above argument shows that the first-best will not be achievable using a Groves scheme. In fact the argument establishes more. Consider *any* three party contract involving B, S, and T, where B, S, T are risk neutral. Then, since B and T (or S and T) cannot lose from merging, this contract must be equivalent to a two party contract involving just B and S. In other words, we may as well focus on two party contracts from

Moreover, one would expect this externality to lead to under-investment, as in Proposition 4: one party's private gain from additional investment is less than the social gain since it does not take into account the benefit accruing to the other party. This under-investment result is not new; it can be found in the work of Williamson (see, e.g., Williamson (1985)) and Klein, Crawford, and Alchian (1978), and more recently in Grout (1984). However, we believe that Proposition 4 is the first under-investment result where there has been an analysis of the precise limitations on the feasible ex post divisions of surplus implied by contractual incompleteness. The connection between an ex ante incomplete contract and the ex post revision of its terms is clearly of central importance, and it is in this respect that we see our paper as contributing to the literature on under-investment.

Risk Aversion

Throughout the paper we have concentrated on the role that contracts play in facilitating trade where there are specific investments and the parties are risk neutral. If the agents are risk averse, then clearly the nature of an optimal contract changes. One case which we have explored is that of pure risk sharing, with no investments. In this case it turns out that when messages are verifiable it is possible to achieve first-best. We briefly explain why.

If the buyer's and seller's von Neumann-Morgenstern utilities are $B(\cdot)$ and $S(\cdot)$ respectively, then optimal risk sharing requires some constant no-trade price p_0 when $v < c$, and some trade price $p_1(v, c)$ satisfying the Borch condition:

$$\frac{B'(v-p_1(v,c))}{B'(-p_0)} = \frac{S'(p_1(v,c)-c)}{S'(p_0)} \qquad \text{for all } v \geq c.$$

This implies that there is some function $\phi(\cdot)$, whose derivative lies between 0 and 1, such that $v - p_1(v, c)$ equals $\phi(v-c)$ and $p_1(v, c) - c$ equals $v - c - \phi(v - c)$. That is, each party's net payoff is a function only of total surplus, $\max\{v - c, 0\}$. All three conditions of Proposition 2 are satisfied by $p_1(v, c)$ and p_0. So these

the beginning. A similar conclusion may hold even if B and S are risk averse; see Appendix A of Hart and Moore (1985).

We should stress that the arguments just given depend on the fact that no third party can be found who is "honest" in the sense of being unwilling to participate in side-contracting. Of course, in practice there are arbitration services who live off their reputation for honesty, or for dealing equitably. We therefore do not wish to claim that our restriction to bilateral contracts always applies. However, there is scope to at least partially neutralize a three party contract, even if the third party is honest, because the buyer and seller could collude against the third party.

Given the assumption that third parties are corruptible, the reader may wonder whether it is reasonable to suppose that the courts are not. One justification is that, given the possibility of appeal, several courts may be involved in judging the case and it may be difficult for one party to bribe them all (in contrast, there is a single designated third party).

prices can be achieved using verifiable messages even though contingent contracts cannot be written directly.

6. CONCLUSIONS

In this paper, we have studied a situation in which two contracting parties are forced to write an incomplete contract because of their inability to specify the state of the world in sufficient detail that an outsider can verify whether it has occurred. We have explored whether the parties can make up for this incompleteness to some extent by building into their contract a mechanism for revising the terms of trade as each party receives information about benefits and costs. It has emerged that the divisions of ex post surplus which can be achieved are very sensitive to the characteristics of the communication mechanism at the parties' disposal—in particular, whether the parties' messages are verifiable or not.

For the case where the parties are risk neutral and must undertake relationship-specific investments, we showed that the parties will not generally be able to sustain efficient investment levels even if messages are verifiable. Furthermore, given special—although not implausible—assumptions about the stochastic technology, we were able to confirm the idea that the second-best outcome would involve under-investment.

Also, for the case where the parties are risk averse, but where there are no specific investments, we showed that it is possible to achieve the first-best if the parties' messages are verifiable.

A natural question to ask is whether mechanisms of the sort that we have described are found in practice. It is very common for long-term contracts to contain formulae linking future terms of trade to some objective industry price or cost index, or to actual cost through a cost plus arrangement. Our mechanism is rather different, however, in that it involves one or both parties having a direct influence over the terms of trade (the mechanism could, of course, easily be supplemented by the use of external indexes or cost plus arrangements). It is worth noting that, in his interesting study of long-term contracts involving coal suppliers and electricity generating plants, Joskow (1985) discusses a case of a contract which gave the coal supplier an option to switch from an indexed arrangement to a cost plus arrangement on six months notice. This is a special case of the mechanism we consider (in general, both parties will have some choice over the price schedule), although it is also consistent with certain asymmetric information mechanisms (see, e.g., Riordan (1984)).

One aspect of our analysis to which attention should be drawn is that in equilibrium the parties never have to rescind their initial contract and write a new one (see the comments following Proposition 1). The reason for this is that, since the parties have unlimited ability to conceive of all the possible benefit/cost situations—that is, (v, c) pairs—any renegotiation can be anticipated and built into the revision process in the original contract. In reality, of course, parties frequently write a limited term contract, with the intention of renegotiating this when it comes to an end. In order to understand this phenomenon, it seems likely that one will have to drop the assumption that the parties have unbounded rationality. It goes without saying that this is a vital—if forbiddingly difficult—topic for future research.

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Manuscript received April, 1985; final revision received June, 1987.

APPENDIX A

Proof of Proposition 1

The full sequence of moves that follows the realization of v and c is laid out in Figure 3, a schematic game tree which is not meant to be formal. (It must be borne in mind, of course, that even this is not the *entire* game played by the buyer and seller: there is an earlier stage, prior to date 1, at which the players choose their investments β and σ respectively.)

Two matters need to be resolved before this can be studied as a formal game. First, what is the sequence of moves at the dispute stage? In Figure 3 this has been depicted as a simultaneous, single-stage subgame; each party is free to reveal (with or without his own signature appended) none, some, or all of the messages that the other sent to him between dates 1 and 2. Actually, the precise specification of this (presumed finite) dispute subgame is immaterial—provided that each party has the opportunity to reveal the messages he received—because, as we shall see, there is a unique (subgame perfect) equilibrium outcome which is unaffected by the order in which a dispute is resolved.

Second, we need to specify how a court rules when faced with a set of messages presented by each of the disputing parties. It is tempting merely to say that a court rules "logically." But the matter is not necessarily that straightforward; the evidence may be mutually contradictory. For example, what should a court decide if trade has occurred at date 2 and it is presented with two new contracts which are both jointly signed, and which both rescind the old date 0 contract, but which specify *different* trading prices? Another problem is that individual messages need not even be internally coherent: how should a court construe such a message? Once again, as we shall show, there is a unique equilibrium outcome which is independent of the way in which the courts resolve ambiguous cases. All that is required is that in those cases where there is no ambiguity, a court's judgement be logical. Simply for clarity, however, we shall assume that in all other cases, a court will enforce the old contract and ignore conflicting evidence.

Stage	Buyer's move	Seller's move
date 1	learn (v, c)	learn (v, c)
day 1	send message(s) $\boldsymbol{m}_b(1)$	send message(s) $\boldsymbol{m}_s(1)$
day 2	receive message(s) $\boldsymbol{m}_s(1)$ send message(s) $\boldsymbol{m}_b(2)$	receive message(s) $\boldsymbol{m}_b(1)$ send message(s) $\boldsymbol{m}_s(2)$
⋮	⋮	⋮
day d	receive message(s) $\boldsymbol{m}_s(d-1)$ send message(s) $\boldsymbol{m}_b(d)$	receive message(s) $\boldsymbol{m}_b(d-1)$ send message(s) $\boldsymbol{m}_s(d)$
date 2	receive message(s) $\boldsymbol{m}_s(d)$ trade/not trade payment	receive message(s) $\boldsymbol{m}_b(d)$ trade/not trade payment
subsequent date	choose from $\{\boldsymbol{m}_s(1), \ldots, \boldsymbol{m}_s(d)\}$ ($\pm$ own signature) to reveal to court	choose from $\{\boldsymbol{m}_b(1), \ldots, \boldsymbol{m}_b(d)\}$ ($\pm$ own signature) to reveal to court

Figure 3.—Game played in Case A following realization of v and c at date 1.

Turning now to Proposition 1, there are four regions to consider: (1) $v < c$; (2) $v \geq \hat{p}_1 - \hat{p}_0 \geq c$; (3) $v \geq c > \hat{p}_1 - \hat{p}_0$; and (4) $\hat{p}_1 - \hat{p}_0 > v \geq c$. We fully dealt with (1) in the text. Here, it is enough to consider just (2) and (3), since (4) is symmetric to (3).

Region (2): $v \geq \hat{p}_1 - \hat{p}_0 \geq c$. Suppose that there are strategies for the buyer and seller which constitute a subgame perfect equilibrium and lead to a trade/price outcome other than $q = 1/p = \hat{p}_1$. Further, suppose that the buyer obtains strictly less than $(v - \hat{p}_1)$ net. (The only other possibility, that the seller obtains strictly less than $(\hat{p}_1 - c)$ net, can be dealt with by a symmetric argument.) Then consider the following alternative strategy for the buyer:

Send no messages up to and including day $d-1$. In the meantime the seller will have sent, according to his strategy, messages $\boldsymbol{m}_s(1), \boldsymbol{m}_s(2), \ldots, \boldsymbol{m}_s(d-1)$, say. Moreover the buyer, knowing the seller's strategy (the Nash assumption), can anticipate the message(s), $\boldsymbol{m}_s(d)$, the seller will send on day d. (If the seller is playing a mixed strategy on day d, then let $\boldsymbol{m}_s(d)$ be the support of the distribution of messages from which the seller selects.) On day d, the buyer sends the following message to the seller:

> "Dear seller, I propose we rescind all the new contracts, and any amendments to the old contract, contained in your messages $\boldsymbol{m}_s(1), \boldsymbol{m}_s(2), \ldots, \boldsymbol{m}_s(d)$. If you agree to this, sign here and retain. Yours, buyer."

Thereafter, the buyer's strategy is to trade at date 2, and not to reveal any of the messages $\boldsymbol{m}_s(1), \boldsymbol{m}_s(2), \ldots, \boldsymbol{m}_s(d)$ to a court.

Faced with the buyer's letter at date 2, the seller can either trade or not. If on the one hand trade does occur, then the final price must be $\hat{p}_1$. This is because a different (expected) price, p_1', say, could only arise if in a dispute, with some positive probability, the buyer were to reveal some or all of the messages $\boldsymbol{m}_s(1), \boldsymbol{m}_s(2), \ldots, \boldsymbol{m}_s(d)$ to a court, and the seller were to reveal nothing. But this is not an equilibrium, since if $p_1' > \hat{p}_1$ the buyer could always withhold all of the seller's messages and be guaranteed $\hat{p}_1$, whereas if $p_1' < \hat{p}_1$ the seller could always produce the buyer's message and be guaranteed $\hat{p}_1$. (There is an important subtlety here, which deserves proper consideration; see the "Remark" at the end of this Appendix.) If on the other hand trade does not occur, then by a similar line of reasoning the final price must be $\hat{p}_0$. Since $\hat{p}_1 - c \geq \hat{p}_0$, the seller will trade in equilibrium. (If the seller is indifferent between trading and not (i.e., if $\hat{p}_1 - c = \hat{p}_0$), then there can never be an equilibrium without trade, because the buyer would offer a trade price of slightly more than $\hat{p}_1$ in his message on day d.)

The final outcome of the buyer's deviation, then, is trade at price $\hat{p}_1$. The buyer is strictly better off, and so the previous pair of strategies cannot have constituted a subgame perfect equilibrium.

Region (3): $v \geq c > \hat{p}_1 - \hat{p}_0$. Again, suppose that there are strategies for the buyer and seller which constitute a subgame perfect equilibrium and lead to a trade/price outcome other than $q = 1/p = \hat{p}_0 + c$. Now the seller can unilaterally obtain at least $\hat{p}_0$ net by not sending any messages between dates 1 and 2, not trading at date 2, and after date 2 not revealing any messages that the buyer may have sent. Therefore it must be the case that the buyer is receiving strictly less than $(v - \hat{p}_0 - c)$ net. Consider the following alternative strategy for the buyer, which is similar to that described (more fully) in (2) above:

Send no messages on days 1 through $d-1$, and on day d send the following message to the seller:

> "Dear seller, I propose we rescind not only the old contract, but also all the new contracts contained in your messages $\boldsymbol{m}_s(1), \boldsymbol{m}_s(2), \ldots, \boldsymbol{m}_s(d)$. I propose we set $p_1 = \hat{p}_0 + c$ and $p_0 = \hat{p}_0$. If you agree, sign here and retain. Yours, buyer."

Thereafter, the buyer's strategy is to trade at date 2, and not to reveal any of the messages $\boldsymbol{m}_s(1), \boldsymbol{m}_s(2), \ldots, \boldsymbol{m}_s(d)$ to a court.

Faced with the buyer's letter at date 2, the seller can either trade or not. If on the one hand trade does occur, then the final price must be $\hat{p}_0 + c$. This is because a different (expected) price, p_1', say, could only arise if in a dispute, with some positive probability, the seller were to reveal nothing. But this is not an equilibrium, since if $p_1' > \hat{p}_0 + c$ then the buyer could always withhold all of the seller's messages and be guaranteed an expected price in the range $[\hat{p}_1, \hat{p}_0 + c]$, and if $p_1' < \hat{p}_0 + c$, then the seller could always produce the buyer's message and be guaranteed $\hat{p}_0 + c$. (Again, there are certain subtleties; analogous arguments to those given below (under "Remark") apply here.) If on the other hand trade does not occur, then by a similar line of reasoning the final price must be $\hat{p}_0$. So the seller will trade in equilibrium. (Although the seller is indifferent between trading and not, there can never be an equilibrium without trade, because the buyer would offer a trade price of slightly more than $\hat{p}_0 + c$ in his message on day d.)

The final outcome of the buyer's deviation, then, is trade at price $\hat{p}_0 + c$. The buyer is strictly better off, and so the previous pair of strategies cannot have constituted a subgame perfect equilibrium.

We have so far proved that in regions (2) and (3)—and therefore, by symmetry, in region (4) too—a division of surplus other than that given in Proposition 1 cannot be a subgame perfect equilibrium outcome. It is clear that the date 0 contract given in the text, in which the buyer has the power to raise $\hat{p}_1$ and the seller has the power to lower $\hat{p}_1$, does indeed lead to trading at the prices specified in the Proposition.

REMARK: Since the parties are free to send any message whatsoever, it is tempting to think that there may be rather sophisticated ways for them to increase their share of the surplus. In fact this is not the case.

Consider the seller, for example, in region (2). Suppose he sends just one message, on day d: $\boldsymbol{m}_s(d)$. The message $\boldsymbol{m}_s(d)$ is as follows:

> "Dear buyer, I propose we rescind the old contract. Moreover, unless we agree to a new contract in which $p_1 = \hat{p}_0 + v$ and $p_0 = \hat{p}_0$, I propose that we rescind all other new contracts, and set $p_1 = \hat{p}_0 + c - \varepsilon$ (where $\varepsilon > 0$) and $p_0 = \hat{p}_0$. If you agree to this, sign here and retain. Yours, seller."

The fact that the buyer possesses this letter at date 2 (and has no opportunity to prove to the seller that he has got rid of it—e.g., by sending it back) poses him with a potential problem. If he sends no messages, then the seller will not trade because once trade has occurred, the buyer will then have an incentive to reveal the letter in court and the price will be reduced from $\hat{p}_1$ to $\hat{p}_0 + c - \varepsilon$, thus giving the seller a net loss of ε (compared with no trade). From both players' perspectives, the letter $\boldsymbol{m}_s(d)$ threatens the overall gains from trade, $(v - c)$; but only the buyer can neutralize the threat. One route the buyer could follow is simply to acquiesce, by simultaneously writing to the seller (at day d):

> "Dear seller, I propose we rescind the old contract, and set $p_1 = \hat{p}_0 + v$ and $p_0 = \hat{p}_0$. If you agree to this, sign here and retain. Yours, buyer."

This successfully neutralizes the threat—it meets the conditional clause of the seller's letter—but of course it leaves the buyer with none of the surplus. Is this pair of messages an equilibrium, contrary to Proposition 1? We argue not, because the buyer could instead send the following message, $\boldsymbol{m}_b(d)$ (the form of which is given above in the analysis for region (2)):

> "Dear seller, I propose we rescind the new contract contained in your letter $\boldsymbol{m}_s(d)$. If you agree, sign here and retain. Yours, buyer."

The crucial question is: what would a court decide if faced with messages $\boldsymbol{m}_s(d)$ *and* $\boldsymbol{m}_b(d)$, each signed by both parties? On the one hand, the contract in $\boldsymbol{m}_s(d)$ (given that the conditional clause is not satisfied) rescinds *all* other contracts—ostensibly including that in $\boldsymbol{m}_b(d)$. But on the other hand, the contract in $\boldsymbol{m}_b(d)$ *specifically* rescinds that in $\boldsymbol{m}_s(d)$. Our view is that the court would enforce the contract in $\boldsymbol{m}_b(d)$. For if the court did not, then it would be tantamount to admitting the possibility of contracts which can never be rescinded—it would suffice to include a clause like "we agree to rescind *all* other contracts." But our basic assumption (**) rules this out.

Buyer \ Seller	w_1	w_2	w_3		w_{t+1}
w_1	$(\tilde{p}_1(w_1)-v_1, \tilde{p}_1(w_1))$				$(\tilde{p}_1(w_1)-c_1, \tilde{p}_1(w_1))$
w_2		$(\tilde{p}_1(w_2)-v_2, \tilde{p}_1(w_2))$			$(\tilde{p}_1(w_2)-c_2, \tilde{p}_1(w_2))$
w_3			$(\tilde{p}_1(w_3)-v_3, \tilde{p}_1(w_3))$		$(\tilde{p}_1(w_3)-c_3, \tilde{p}_1(w_3))$
w_{t+1}	$(\tilde{p}_1(w_1)-v_1, \tilde{p}_1(w_1))$	$(\tilde{p}_1(w_2)-v_2, \tilde{p}_1(w_2))$	$(\tilde{p}_1(w_3)-v_3, \tilde{p}_1(w_3))$		$(\tilde{p}_0, \tilde{p}_0)$

FIGURE 4

APPENDIX B

PROOF OF SECOND PART OF PROPOSITION 2

Take all distinct pairs (v, c) which have a positive joint probability, and for which trading is efficient, i.e., $v \geq c$. Suppose there are $t \geq 1$ such pairs. Then number them

$$w_1 = (v_1, c_1), \qquad w_2 = (v_2, c_2), \ldots, \qquad w_t = (v_t, c_t),$$

where w is just a shorthand for (v, c). Finally, let w_{t+1} stand for all (v, c) pairs (if any) which have a positive joint probability and $v < c$.

We choose the payoffs of the message game in such a way that in equilibrium both parties want to tell the truth. Note that we cannot punish the parties for "disagreeing" about (v, c) since, in the absence of a third party, one party's punishment is another's reward.

The construction of the game is illustrated in Figure 4. The diagonal elements, for $v \geq c$, consist of a trading price equal to the desired one, $\tilde{p}_1(v, c)$, and a nontrading price chosen to ensure that trade occurs (we have selected $p_0 = \tilde{p}_1(v, c) - v$, but any $p_0 = \tilde{p}_1(v, c) - k$ where $c \leq k \leq v$ would do). The final diagonal element has the nontrading price equal to the desired one, $\tilde{p}_0$ (since trade never occurs when $v < c$, the trading price is irrelevant—here we have set it at $\tilde{p}_0$). By assumption, $\tilde{p}_1(v, c)$ and $\tilde{p}_0$ satisfy conditions (1)–(3) of the Proposition.

The off-diagonal elements are a bit more complicated. They are indicated in the diagram for the case where one party announces w_{t+1}. We now describe how they are determined in the case where one party announces w_i and the other w_j, where $i, j < t+1$, $i \neq j$.

Consider the sub-box (or subgame) corresponding to the announcements w_i, w_j. Without loss of generality, suppose $\tilde{p}_1(w_j) \geq \tilde{p}_1(w_i)$. There are three cases:

Case (i): $\tilde{p}_1(w_j) = \tilde{p}_1(w_i)$. Then choose

$$\left(p_0^{ij}, p_1^{ij}\right) = \left(p_0^{ji}, p_1^{ji}\right) = \left(\tilde{p}_0, \tilde{p}_1(w_j)\right).$$

Case (ii): $\tilde{p}_1(w_j) > \tilde{p}_1(w_i)$ and $v_j - v_i \geq c_j - c_i$. Then choose

$$\left(p_0^{ij}, p_1^{ij}\right) = \left(\tilde{p}_1(w_j) - v_j, \tilde{p}_1(w_j)\right),$$

$$\left(p_0^{ji}, p_1^{ji}\right) = \left(\tilde{p}_1(w_i) - v_i, \tilde{p}_1(w_i)\right).$$

Case (iii): $\tilde{p}_1(w_j) > \tilde{p}_1(w_i)$ and $v_j - v_i < c_j - c_i$. Then choose

$$\left(p_0^{ij}, p_1^{ij}\right) = \left(\tilde{p}_1(w_i) - c_i, \tilde{p}_1(w_i)\right),$$

$$\left(p_0^{ji}, p_1^{ji}\right) = \left(\tilde{p}_1(w_j) - c_j, \tilde{p}_1(w_j)\right).$$

We have now described how all the payoffs of the game are determined (if $\tilde{p}_1(w_j) < \tilde{p}_1(w_i)$, reverse i, j in cases (ii) and (iii) above). It remains to show that given any realized $(v_i, c_i) = w_i$, each party will tell the truth, i.e. send the message w_i—so that the price $\tilde{p}_1(w_i)$, together with trade, is implemented if $v_i \geq c_i$, and the price $\tilde{p}_0$, together with no trade, is implemented if $v_i < c_i$.

Suppose first the realization is such that $v < c$. Then no trade occurs whatever messages are sent. If the buyer announces w_j, $j < t+1$, while the seller announces w_{t+1}, the buyer pays price $\tilde{p}_1(w_j) - c_j \geq \tilde{p}_0$ by condition (3) of the Proposition. Hence a deviation by the buyer from the strategies (w_{t+1}, w_{t+1}) is not profitable. On the other hand, if the seller announces w_j, $j < t+1$, while the buyer announces w_{t+1}, the seller receives $\tilde{p}_1(w_j) - v_j \leq \tilde{p}_0$ by condition (3). Hence a deviation by the seller is also not profitable. It follows that (w_{t+1}, w_{t+1}) is a Nash equilibrium when $v < c$. (Note that there may be other Nash equilibria; however since the game is zero-sum, they are all equivalent.)

Suppose next the realization is such that $v_i \geq c_i$. Consider first whether the buyer wants to deviate from "truth-telling," given that he expects the seller to announce w_i. If the buyer announces w_{t+1}, the price pair will be $(\tilde{p}_1(w_i) - v_i, \tilde{p}_1(w_i))$, and so, since $v_i \geq c_i$, trade will occur at price $\tilde{p}_1(w_i)$, which is also the ruling price if the buyer tells the truth. So a deviation to w_{t+1} is not profitable for the buyer. What about a deviation to w_j, where $j < t+1$? Then Figure 5 applies, and the price pair is (p_0^{ji}, p_1^{ji}). To see that such a deviation is unprofitable, we separately consider the following cases:

Case (a): $\tilde{p}_1(w_j) = \tilde{p}_1(w_i)$. Trade occurs at price $\tilde{p}_1(w_j)$, and so the buyer gains nothing.

Case (b): $\tilde{p}_1(w_j) > \tilde{p}_1(w_i)$ and $v_j - v_i \geq c_j - c_i$. Trade occurs at price $\tilde{p}_1(w_i)$, and so the buyer gains nothing.

Case (c): $\tilde{p}_1(w_j) > \tilde{p}_1(w_i)$ and $v_j - v_i < c_j - c_i$. From condition (2) of the Proposition, $c_j - c_i \geq \tilde{p}_1(w_j) - \tilde{p}_1(w_i) > 0$, so the seller wants to trade at prices $(\tilde{p}_1(w_j) - c_j, \tilde{p}_1(w_j))$. But the buyer may not (he won't if $v_i < c_j$). If the buyer does want to, the trading price will be $\tilde{p}_1(w_j)$, which exceeds $\tilde{p}_1(w_i)$, and so he will not have gained by his deviation. If the buyer does not want to trade at prices $(\tilde{p}_1(w_j) - c_j, \tilde{p}_1(w_j))$, the contract will be renegotiated and the trading price will be $\tilde{p}_1(w_j) - c_j + v_i$. But from condition (3), this amount is at least $\tilde{p}_1(w_i)$. Hence the buyer's deviation is unprofitable.

Case (d): $\tilde{p}_1(w_i) > \tilde{p}_1(w_j)$ and $v_i - v_j \geq c_i - c_j$. Trade occurs at price $\tilde{p}_1(w_i)$, and so the buyer gains nothing.

Seller

		w_i	w_j
Buyer	w_i	$(\tilde{p}_1(w_i)-v_i, \tilde{p}_1(w_i))$	(p_0^{ij}, p_1^{ij})
	w_j	(p_0^{ji}, p_1^{ji})	$(\tilde{p}_1(w_j)-v_j, \tilde{p}_1(w_i))$

FIGURE 5

Case (e): $\tilde{p}_1(w_i) > \tilde{p}_1(w_j)$ and $v_i - v_j < c_i - c_j$. From condition (2) of the Proposition, $c_i - c_j \geq \tilde{p}_1(w_i) - \tilde{p}_1(w_j) > 0$, so the seller will not trade at prices $(\tilde{p}_1(w_j) - c_j, \tilde{p}_1(w_j))$. The contract will be renegotiated and the trading price will be $\tilde{p}_1(w_j) - c_j + c_i$, which (again by condition (2)) is at least $\tilde{p}_1(w_i)$. Hence the buyer's deviation is unprofitable.

We have established that in all cases, if the seller announces the truth, the buyer can do no better than announce the truth too. A similar argument shows that it does not pay the seller to deviate from the truth, if the buyer is not going to. Hence truth-telling is a Nash equilibrium if $v \geq c$ (again there may be other Nash equilibria, but they are all equivalent).

This proves Proposition 2. *Q.E.D.*

APPENDIX C

PROOF OF PROPOSITION 4

Proposition 2 specifies what prices can be implemented when messages are verifiable. It will be helpful to simplify the notation a little.

First, for all $v_i \geq c_j$, define

$$p_{ij} \equiv p_1^*(v_i, c_j) - p_0^*.$$

Then, from parts (1) and (3) of Proposition 2, we know

$$p_{i,j+1} \leq p_{ij} \leq p_{i+1,j} \qquad \text{(monotonicity)}$$

and

$$c_j \leq p_{ij} \leq v_i \qquad \text{(voluntary trade)}.$$

Secondly, for all i and j, define

$$\Delta\pi_i \equiv \pi_i^+ - \pi_i^-$$

and

$$\Delta\rho_j \equiv \rho_j^+ - \rho_j^-.$$

Then $\Delta\pi_i/\pi_i(\beta)$ and $\Delta\rho_j/\rho_j(\sigma)$ are increasing in i and j. Notice that these imply first-order stochastic dominance.

Thirdly, in what follows, let Σ denote

$$\sum_{i=1}^{I} \sum_{\substack{j=1 \\ v_i \geq c_j}}^{J}.$$

The buyer's net gain from marginally increasing β is

$$\sum \Delta\pi_i \rho_j(\sigma)[v_i - p_{ij}] - h_b'(\beta).$$

The first term is bounded. It is also nonnegative—using stochastic dominance and the fact that $v_i - p_{ij}$ is nondecreasing in i (from Proposition 2(2)). So it follows from condition (3) of Proposition 4 that a necessary and sufficient condition for the buyer's optimal choice of β in $[0,1]$ is

$$\text{(C.1)} \qquad \sum \Delta\pi_i \rho_j(\sigma)[v_i - p_{ij}] - h_b'(\beta) = 0.$$

Likewise, the seller's choice of σ can be summarized by

$$\text{(C.2)} \qquad \sum \pi_i(\beta) \Delta\rho_j [p_{ij} - c_j] - h_s'(\sigma) = 0.$$

Consider the following relaxed program (RP):

$$\text{Maximize} \sum \pi_i(\beta)\rho_j(\sigma)[v_i - c_j] - h_b(\beta) - h_s(\sigma)$$

subject to

$$\text{(C.3)} \qquad \sum \Delta\pi_i \rho_j(\sigma)[v_i - p_{ij}] - h_b'(\beta) \geq 0,$$

$$\text{(C.4)} \qquad \sum \pi_i(\beta) \Delta\rho_j [p_{ij} - c_j] - h_s'(\sigma) \geq 0,$$

$$\beta, \sigma \geq 0$$

and monotonicity and voluntary trade.

(RP) is "relaxed" in two respects. First, the equalities (C.1) and (C.2) have been replaced by inequalities. This is just a technical device: in Lemma 3 below, their respective (nonnegative) Kuhn-Tucker multipliers γ and θ will be shown to be positive at an optimum, implying that the inequality constraints are binding. Secondly, the restriction on $p_1^*(v, c)$ in Proposition 2(2) has been omitted. The reason for this is that, as we shall see, the trading prices which solve (RP) satisfy this restriction anyway.

Note that the level of p_0^* is left undetermined in (RP); this is because it is equivalent to a transfer payment at date 0 which ensures that the contract is worth $\bar{U}$ to the seller.

The necessary first-order condition for β is

$$\sum \Delta\pi_i \rho_j(\sigma)[v_i - c_j] - h_b'(\beta) - \gamma h_b''(\beta) + \theta \sum \Delta\pi_i \Delta\rho_j [p_{ij} - c_j] \leq 0,$$

with equality if $\beta > 0$. Using (C.3), we see that this implies

$$\text{(C.5)} \qquad \sum \Delta\pi_i \rho_j(\sigma)[p_{ij} - c_j] - \gamma h_b''(\beta) + \theta \sum \Delta\pi_i \Delta\rho_j [p_{ij} - c_j] \leq 0.$$

Likewise the first-order condition for σ implies

$$\text{(C.6)} \qquad \sum \pi_i(\beta) \Delta\rho_j [v_i - p_{ij}] - \theta h_s''(\sigma) + \gamma \sum \Delta\pi_i \Delta\rho_j [v_i - p_{ij}] \leq 0.$$

The proof of the Proposition proceeds via the following three Lemmata.

LEMMA 1: *At a solution to (RP), if $\theta > 0$, then for $j < J$ and $v_i \geq c_j$,*

$$p_{ij} = \max\{p_{i,j+1}, c_j\},$$

and if $\gamma > 0$, then for $i < I$ and $v_i \geq c_j$,

$$p_{ij} = \min\{p_{i+1,j}, v_i\}.$$

PROOF: By symmetry, we need only prove the first half of the Lemma. Suppose it is not true: for some $v_i \geq c_j$,

$$p_{ij} = k^+ \qquad \text{and} \qquad p_{i,j+1} = k^- \qquad \text{where } k^+ \text{ exceeds } k^- \text{ and } c_j.$$

Let t be the minimum τ satisfying $p_{\tau j} = k^+$ and $v_\tau \geq c_j$. And let T be the maximum τ satisfying $p_{\tau, j+1} = k^-$. Then monotonicity and voluntary trade imply that $p_{\tau, j+1} < p_{\tau j} \leq v_\tau$ and $p_{\tau j} \geq k^+ > c_j$ for all τ in $\{t, \ldots, T\}$. And so without violating monotonicity or voluntary trade, we can, for each τ in $\{t, \ldots, T\}$, lower $p_{\tau j}$ by $\varepsilon_j > 0$ and raise $p_{\tau, j+1}$ by $\varepsilon_{j+1} > 0$—where the (small) ε_j, ε_{j+1} are chosen so as not to disturb the left-hand side of (C.3): i.e.

$$\rho_j(\sigma)(-\varepsilon_j) + \rho_{j+1}(\sigma)\varepsilon_{j+1} = 0.$$

The effect on the left-hand side of (C.4) is

$$\sum_{\tau=t}^{T} \pi_\tau(\beta)[\Delta\rho_j(-\varepsilon_j) + \Delta\rho_{j+1}\varepsilon_{j+1}] = \sum_{\tau=t}^{T} \pi_\tau(\beta)\rho_j(\sigma)\varepsilon_j \left[\frac{\Delta\rho_{j+1}}{\rho_{j+1}(\sigma)} - \frac{\Delta\rho_j}{\rho_j(\sigma)}\right]$$

—which is positive. But the fact that we can slacken the constraint (C.4) in this way contradicts $\theta > 0$. *Q.E.D.*

Next a technical Lemma which will be of use later.

LEMMA 2: *If x_i is nondecreasing in i, and y_j is nonincreasing in j, then*

$$\sum_{\substack{i=1 \\ x_i \geq y_j}}^{I} \sum_{j=1}^{J} \Delta\pi_i \Delta\rho_j [x_i - y_j] \geq 0,$$

and the inequality is strict if and only if $x_I - y_J > 0 > x_1 - y_1$.

PROOF: Define $z_{ij} \equiv \max\{0, x_i - y_j\}$. It is straightforward to show that, for $i < I$ and $j < J$,

$$z_{i+1, j+1} - z_{i, j+1} - z_{i+1, j} + z_{ij} \geq 0, \tag{C.7}$$

with equality iff either $x_{i+1} \leq y_{j+1}$ or $x_i \geq y_j$.

For each i, define $\xi_i \equiv \sum_{j=1}^{J} \Delta\rho_j z_{ij}$.

Take a particular $i < I$. Now

$$\xi_{i+1} - \xi_i = \sum_{j=1}^{J} \Delta\rho_j [z_{i+1, j} - z_{ij}],$$

which from (C.7) and stochastic dominance is nonnegative, and zero iff either $x_{i+1} \leq y_{j+1}$ or $x_i \geq y_j$ for all $j < J$—i.e. iff either $x_{i+1} \leq y_J$ or $x_i \geq y_1$. Therefore, again from stochastic dominance, $\sum_{i=1}^{I} \Delta\pi_i \xi_i \geq 0$, with equality iff either $x_{i+1} \leq y_J$ or $x_i \geq y_1$ for all $i < I$—i.e. iff either $x_I \leq y_J$ or $x_1 \geq y_1$. *Q.E.D.*

LEMMA 3: *At a solution to (RP), $\gamma > 0$ and $\theta > 0$.*

PROOF: Suppose not: without loss of generality, suppose $\gamma = 0$.

First we show that $\theta > 0$. For if $\theta = 0$, then from (C.5) and (C.6),

$$\sum \Delta\pi_i \rho_j(\sigma)[p_{ij} - c_j] \leq 0,$$

and

$$\sum \pi_i(\sigma) \Delta\rho_j [v_i - p_{ij}] \leq 0.$$

But monotonicity, voluntary trade, and stochastic dominance together imply that the left-hand sides of these inequalities are nonnegative, and equal zero if and only if (i) $v_I \geq c_j$ ($\Rightarrow p_{ij}$ is independent of i, and equal to c_j if $v_1 < c_j$); *and* (ii) $c_J \leq v_i$ ($\Rightarrow p_{ij}$ is independent of j, and equal to v_i if $c_1 > v_i$). Either $v_1 < c_J$ or $v_1 \geq c_J$. Therefore, either $p_{IJ} = c_J$ or $p_{IJ} = p_{1J} = v_1$. But a symmetric argument shows that either $p_{IJ} = v_I$ or $p_{IJ} = c_1$. However this contradicts $I, J \geq 2$ and assumption (4) of the Proposition. Hence $\theta > 0$.

Consider the left-hand side of (C.5). Monotonicity, voluntary trade, and stochastic dominance together imply that the first term is nonnegative. The second term is zero, since $\gamma = 0$. Hence the third term must be nonpositive. But, since $\theta > 0$, Lemma 1 tells us that, for those $v_i \geq c_j$, either $p_{ij} = c_j$ or p_{ij} equals some x_i, say, which is independent of j and (from monotonicity) nondecreasing in i. Hence Lemma 2 applies (setting $y_j \equiv c_j$), and the third term of the left-hand side of (C.5) is nonnegative, and zero only if $p_{ij} \equiv c_j$ for all $v_i \geq c_j$. But this last cannot be the case, since it would mean $p_{IJ} < \min\{v_I, p_{I, J-1}\}$ and therefore a first-order condition for p_{IJ}: $\theta\pi_I(\beta)\Delta\rho_J \leq 0$—which contradicts $\theta, \Delta\rho_J > 0$. *Q.E.D.*

Lemmata 1 and 3 together imply that in a second-best contract, the trading prices have the form given in Figure 2. These can be achieved with a simple two-price contract $(p_0, p_0 + k)$ without messages, as claimed in Proposition 4. Note that the omitted restriction on trading prices given in Proposition 2(2) is satisfied.

For clarity, denote the second-best levels of β, σ by $\hat{\beta}, \hat{\sigma}$. It remains to show that $\hat{\beta} < \beta^*$ and $\hat{\sigma} < \sigma^*$. Define

$$G(\beta, \sigma) \equiv \sum \pi_i(\beta)\rho_j(\sigma)[v_i - c_j],$$

$$X(\beta, \sigma) \equiv \sum \pi_i(\beta)\rho_j(\sigma)[v_i - p_{ij}],$$

$$Y(\beta, \sigma) \equiv \sum \pi_i(\beta)\rho_j(\sigma)[p_{ij} - c_j].$$

Then $G_1(\beta^*, \sigma^*) = h_b'(\beta^*)$ and $G_2(\beta^*, \sigma^*) = h_s'(\sigma^*)$. Also G_1, X_1, Y_1 are independent of β, and G_2, X_2, Y_2 are independent of σ. Monotonicity, voluntary trade, and stochastic dominance together imply $G_1 \geq X_1$, $X_2 \geq 0$, $G_2 \geq Y_2$, and $Y_1 \geq 0$. Finally, $G_{12} = \sum \Delta\pi_i \Delta\rho_j [v_i - c_j] > 0$ from Lemma 2.

Suppose $\hat{\sigma} \leq (<)\sigma^*$. Then

$$h_b'(\beta^*) = G_1(\beta^*, \sigma^*) \geq (>) G_1(\beta^*, \hat{\sigma}) = G_1(\hat{\beta}, \hat{\sigma}) \geq X_1(\hat{\beta}, \hat{\sigma}) = h_b'(\hat{\beta}).$$

Thus $\hat{\beta} \leq (<)\beta^*$. Since we have shown, in the first part of the proof of Lemma 3, that $(G_1 - X_1, G_2 - Y_2) \neq (0,0)$, it follows that $(\hat{\beta}, \hat{\sigma}) \neq (\beta^*, \sigma^*)$. By symmetry, then, either $\hat{\beta} < \beta^*$ and $\hat{\sigma} < \sigma^*$, or $\hat{\beta} > \beta^*$ and $\hat{\sigma} > \sigma^*$. We now use revealed preference to rule out the latter possibility (we are grateful to Steven Matthews for this argument).

Since (β^*, σ^*) is first-best,

$$G(\hat{\beta}, \hat{\sigma}) - G(\beta^*, \sigma^*) < h_b(\hat{\beta}) - h_b(\beta^*) + h_s(\hat{\sigma}) - h_s(\sigma^*)$$

which in turn is no more than $X(\hat{\beta}, \hat{\sigma}) - X(\beta^*, \hat{\sigma}) + Y(\hat{\beta}, \hat{\sigma}) - Y(\hat{\beta}, \sigma^*)$ since $\hat{\beta}$ is the buyer's choice if $\sigma = \hat{\sigma}$ and $\hat{\sigma}$ is the seller's choice if $\beta = \hat{\beta}$. But $X(\hat{\beta}, \hat{\sigma}) + Y(\hat{\beta}, \hat{\sigma}) = G(\hat{\beta}, \hat{\sigma})$, and so

$$G(\beta^*, \sigma^*) > X(\beta^*, \hat{\sigma}) + Y(\hat{\beta}, \sigma^*). \tag{C.8}$$

Now, using $X_2 \geq 0$, $Y_1 \geq 0$, if $\hat{\beta} > \beta^*$ and $\hat{\sigma} > \sigma^*$, the right-hand side of (C.8) is not less than $X(\beta^*, \sigma^*) + Y(\beta^*, \sigma^*) = G(\beta^*, \sigma^*)$. Contradiction. Proposition 4 is proved. *Q.E.D.*

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