2008 Q1B1

firm to produce. There is a unit mass of consumers in this market. A fraction λ of these consumers view the goods as perfect substitutes and simply buy from the firm offering the lowest price provided it doesn't exceed their maximal willingness to pay. The remaining fraction $1-\lambda$ of consumers infer quality from price. They expect that the firm offering the lowest price must be providing an inferior version of the good. Hence, these consumers purchase from whichever firm offers the highest price provided it doesn't exceed their maximal willingness to pay. All consumers have unit demand and a maximal willingness to pay of 1.

Firms compete in this market by simultaneously making price offers. Every consumer sees both price offers and then makes a purchase decision. In the event both firms offer the same price, consumers are split evenly in the market. Everyone is risk-neutral in the model and consumers buy even if the price is exactly equal to their willingness to pay.

a) Write down firm 1's optimization problem.
b) Suppose that $\lambda \leq \frac{1}{2}$. Find a symmetric pricing equilibrium.
c) Suppose that $1 > \lambda > \frac{1}{2}$. Find a symmetric pricing equilibrium.
d) Now suppose that *Consumer Reports* tests the products of both firms and reveals to all consumers that the two firms' products are identical in every respect. Find a symmetric pricing equilibrium. Relative to the case where $\lambda \leq \frac{1}{2}$ what is the value to consumers (in terms of realized consumer surplus) of the information provided by *Consumer Reports*?

Question B.2: Opportunistic sellers

Consider an economy with many identical Buyers that can each engage in a transaction with one of many sellers. For concreteness, imagine that there are more buyers than sellers and that the market for transactions must clear. The transaction can either succeed or fail. Each buyer's value of "success" is 1 and of "failure" is 0.

There are two kinds of Sellers. A proportion $1-\beta$ are "good" and they succeed with probability $p > 0$. A proportion β are "opportunistic" and can choose some effort, $e \in [0,1]$ at a personal cost of $c(e)$ where $c'(0) = 0$, $c'(1) = \infty$ and $c''(e) > 0$ for all $e \geq 0$. The opportunistic types succeed with probability ep.

The economy operates for 2 periods. Sellers live for two periods but a new cohort of buyers is active in each period, and second period buyers can observe the first period outcome of transactions.

Effort and types are not observable to buyers so that in each period buyers who wish to transact with sellers will pay a price for the transaction in advance, after which the service/good is delivered and failure/success is observed.

(a) What level of e would maximize total surplus?
(b) An equilibrium is defined by a price w_1 that buyers pay sellers in the first period, and history contingent prices $w_2(S)$ and $w_2(F)$ (for Success and Failure respectively) that buyers pay sellers in the second period, so that expectations about [illegible] are correct, and sellers best respond to current and future

2008 – Question B1 = Competition w/ Unknown Quality

λ – consumers buy from lowest priced firm (below WTP)
$1-\lambda$ – consumers buy from highest priced firm (below WTP)

Symmetric price offers by 2 firms (A, B); if same price, split market.
Consumers have unit demand, cost of production for both firms is 0.

(a) Write down firm 1's optimization problem:

$$\text{Firm A: } \max_{p_A} \mathbb{1}\{p_A > p_B\} p_A(1-\lambda) + \mathbb{1}\{p_A < p_B\} p_A(\lambda) + \mathbb{1}\{p_A = p_B\} p_A(\tfrac{1}{2}) \quad \text{where } 0 \le p_A \le 1$$

(b) Find a symmetric price equilibrium of $0 < \lambda < \frac{1}{2}$:

Claim: $p_A^* = p_B^* = 1$, and $\pi_A = \pi_B = \frac{1}{2}$

Check for profitable deviation. WLOG focus on firm A. Note that A cannot set $p_A > 1$ because maximum WTP = 1, so no one purchases for $p_A > 1$.
If instead A deviates to $p_A < p_B = 1$, then:

$\pi_A' = \lambda p_A$ and $\pi_B' = (1-\lambda) p_B$

Where $\lambda < \frac{1}{2}$, $p_A < 1 \Rightarrow \pi_A' < \pi_A = \frac{1}{2}$. So deviation not profitable.

(c) Find a symmetric pricing equilibrium if $\frac{1}{2} < \lambda < 1$:

Claim: $p_A^* = p_B^* = \frac{1}{2}$, and $\pi_A = \pi_B = \frac{1}{4}$. WLOG, check firm A for profitable deviation.
(i) if A picks $p_A < p_B = \frac{1}{2}$, then $\pi_A' = p_A \lambda$ and $p_A < \frac{1}{2}, \lambda < 1 \Rightarrow \pi_A' < \pi_A$.
(ii) if A picks $1 \ge p_A > p_B = \frac{1}{2}$, then $\pi_A'' = p_A(1-\lambda)$ and $p_A < 1, (1-\lambda) < \frac{1}{2} \Rightarrow \pi_A'' < \pi_A$.

(2) Suppose Consumer Reports reveals to all consumers that the two firms' products are identical in every respect. Find a symmetric pricing equilibrium:

Now all consumers will purchase the lowest priced product. Given a (marginal) cost of zero, Bertrand price competition drives the price down to $p_A = p_B = 0$. If either firm attempts to deviate to some price $\varepsilon > 0$, the consumers will purchase from the other, cheaper firm. And if both firms attempt to price at $\varepsilon > 0$, then each face an incentive to deviate to some $\varepsilon' < \varepsilon$, which ensures the equilibrium.

What is the value to consumers (in terms of realized consumer surplus) of the information provided by Consumer Reports, relative to the case where $\lambda < 1/2$?

When $\lambda < 1/2$, we showed in part (b) that $p_A = p_B = 1$. Here we have $p_A = p_B = 0$, so the per consumer surplus is 1, which holds across the unit mass of consumers for a total realized consumer surplus of 1.