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BA 279A Problem Set 2 (Rui)

$$\begin{aligned} u_R(x, y) &= -x^2 + y \\ u_i(x) &= -(x - x_i)^2 \quad i \in \{A, B\} \\ w_A > w_B \text{ and } -x_A > \sqrt{w_A - w_B} \end{aligned}$$

Assume $w_A > w_B > 0$

ideal points: $x_A < x_R = 0 < x_B$

(rendomment)

a. Equilibrium:

$$A: (x^*, w_A)$$

$$B: (0, w_B)$$

$$R: (x^*)$$

$$\text{where } x^* = -\sqrt{w_A - w_B}$$

$$\text{since } u_R(x^*, w_A) = u_R(0, w_B)$$

$$-x^{*2} + w_A = w_B$$

$$x^{*2} = w_A - w_B$$

$$-x^* = \sqrt{w_A - w_B}$$

Tie rule, if A and B tie, R will take A's offer

Proof: WTS ^①A \Rightarrow BR to B, ^②B \Rightarrow BR to A, and ^③R is BR to A and B① Given B offers $(0, w_B)$, A best responds by offering (x^*, w_A) Suppose A offers (x, w_A) s.t. $x < x^*$, then

$$u_R(x, w_A) = -x^2 + w_A < u_R(0, w_B) \text{ so R will reject}$$

$$\text{and } u_A(x) = -(0 - x_A)^2 = -x_A^2 < -(x^* - x_A)^2 = u_A(x^*)$$

Suppose A offers (x, w_A) s.t. $x > x^*$, then

$$u_R(x, w_A) = -x^2 + w_A > u_R(0, w_B) \text{ so R will accept}$$

$$\text{but } u_A(x) = -(x - x_A)^2 < -(x^* - x_A)^2 = u_A(x^*)$$

$$\Rightarrow (x^*, w_A) \triangleright A \text{'s BR}$$

② Given A offers (x^*, w_A) , B best responds by offering $(0, w_B)$ Suppose B offers (x, w_B) s.t. $x < 0$, R will accept but $u_B(x) < u_B(0)$.Suppose B offers (x, w_B) s.t. $x > 0$, R will reject and $u_B(x) < u_B(0)$.

$$\Rightarrow (0, w_B) \triangleright B \text{'s BR}$$

③ Given A offers (x^*, w_A) and B offers $(0, w_B)$, R is indifferentso by the tie rule, R responds by accepting A's offer of (x^*, w_A) and the equilibrium policy $\triangleright x^*$.

$$b. -x_A > \sqrt{w_A - w_B} \Rightarrow x_A < -\sqrt{w_A - w_B} \Rightarrow x_A < x^* \text{ so } x^* \in (x_A, 0)$$

so this constraint prevents corner solutions

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c. If the regulator does not get to keep the payments when the rule is overturned, then:

$$u_R(0, w_B) = u_R(x^*, w_A)$$

$$(1-p)(-0^2 + w_B) + p(-q^2) = (1-p)(-x^{*2} + w_A) + p(-q^2)$$

$$x^{*2} = w_A - w_B$$

$$-x^* = \sqrt{w_A - w_B}$$

\Rightarrow same equilibrium as part (a)

If the regulator does get to keep the payments when the rule is overturned, then:

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$$(1-p)w_B + pw_B - pq^2 = (1-p)w_A + pw_A - (1-p)x^{*2} - pq^2$$

$$w_B = w_A - (1-p)x^{*2}$$

$$x^{*2} = \frac{w_A - w_B}{1-p}$$

$$-x^* = \sqrt{\frac{w_A - w_B}{1-p}}$$

\Rightarrow now the equilibrium is

$$A: (x^*, w_A)$$

$$B: (0, w_B)$$

$$R: (x^*) \quad \text{with } x^* = -\sqrt{\frac{w_A - w_B}{1-p}}$$

But now since $x_A < -\sqrt{w_A - w_B}$

and $0 < p < 1$, $-\sqrt{\frac{w_A - w_B}{1-p}} < -\sqrt{w_A - w_B}$, the corner solution

of $A: (x_A, w_A)$ and B only other could bind.

2. $x_f = 0 \quad x_c > 0$

$$u_f = -(x - x_f)^2 = -x^2 = -(p + \omega)^2$$

$$u_c = -(x - x_c)^2 - sk = -(p + \omega - x_c)^2 - sk$$

unrestrictive procedure with low $k \Rightarrow$ open rule with specialization

$$x_c = \frac{1}{16}$$

$$N = 2$$

$$a_0 = 0 \quad \text{and} \quad a_2 = 1$$

$$\text{since } a_i = a_{i-1} + 2_i(1-i)x_c, \quad a_2 = a_1 \cdot 2 + 2 \cdot 2(1-2)\frac{1}{16}$$

$$\Rightarrow a_2 = 2a_1 - \frac{1}{4} = 1 \Rightarrow 2a_1 = \frac{5}{4} \Rightarrow a_1 = \frac{5}{8}$$

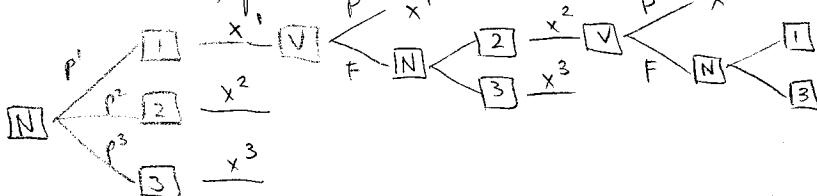
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Interior solution solves

$$\frac{f(x - c_B)}{(1 - f(x - c_B))} = \frac{1}{x - p + c_A}$$

Thus, private benefit instead of cost does not make a difference.

4.

players i, j, k By symmetry, $v_i = v_j = v_k$

To start, if player i is recognized at session t but his proposal was not accepted, then suppose player j was recognized at session $t+1$. Then player j must offer player i or k $\delta v_i = \delta v_k$ and keep $1 - \delta v_i = 1 - \delta v_k$ so k will accept. If player k is recognized, offer i or j $\delta v_i = \delta v_j$.

Then back in session t , player i has to offer player j his continuation value for him to accept, so player i will propose x_i^t to player j .

$$x_i^t = \delta \left(\underbrace{\frac{1}{2}(1 - \delta v_k)}_{\text{recognized}} + \underbrace{\frac{1}{2} \frac{\delta v_j}{2}}_{\text{not recognized}} \right)$$

Then the continuation value

$$v_i = \frac{1}{3} \left(1 - \delta v_i \right) + \frac{2}{3} \left(\frac{\delta v_i}{2} \right)$$

$$= \frac{1}{3}$$

$$x_i^{t'} = 1 - \delta \left(\frac{1}{2} - \frac{\delta}{6} + \frac{1}{2} \frac{\delta}{6} \right)$$

$$= 1 - \delta \left(\frac{1}{2} - \frac{2\delta}{12} + \frac{\delta}{12} \right)$$

$$= 1 - \delta \left(\frac{1}{2} - \frac{\delta}{12} \right)$$

$$= 1 - \frac{1}{2}\delta + \frac{1}{12}\delta^2 = 1 - \frac{8}{3} \left(\frac{3}{2} - \frac{1}{4}\delta \right)$$

From Battin & Fafchamps

$$x_i^t = 1 - \delta(n-1)/2n$$

$$= 1 - \delta/3$$

Since $\max \delta = 1$, $\min \frac{3}{2} - \frac{1}{4}\delta = \frac{5}{4}$, so $x_i^{t'} = 1 - \frac{8}{3} \left(\frac{3}{2} - \frac{1}{4}\delta \right) < 1 - \frac{\delta}{3} = x_i^t$

Thus, the proposer keeps less compared to the standard rule model.

5. a. $q_p < q_{p'}$

If $s=1$, then $-(1-s)\alpha v \geq -s(q_p v + \delta + (1-q_p v))v - (1-s)vq_p(\alpha + \delta)v$

becomes $0 \geq -(q_p v \delta + (1-q_p v))v$

$\Rightarrow \hat{v}(1)=0$, so a type I manager fixes all violations

Since the deviation did not use the relationship between q_p and $q_{p'}$, the result stays the same.

Also, $\hat{v}(1)=0$ by same logic, so a type I manager prefers any violation to be reported privately.

If $s=0$

Type 1: $a_e = q$ (employee does nothing)

then the utility of the type 0 manager is

$$\pi_m = -q_{p'} v (\alpha + \delta) v$$

Type 2: $a_e = p$ (employee privately repays)

If $v < \hat{v}(0)$, manager does not fix violation, then

$$\pi_m = -q_p v (\alpha + \delta) v$$

Since $q_p < q_{p'}$, manager better off when employee repays privately

If $v \geq \hat{v}(0)$, manager fixes violation, then

$$\pi_m = -\alpha v$$

So manager better off when employee repays privately iff

$$-\alpha v \geq -q_{p'} v (\alpha + \delta) v$$

$$\alpha \leq q_{p'} v (\alpha + \delta) v$$

$$v \geq \frac{\alpha}{q_{p'} v (\alpha + \delta)}$$

From Lemma 1, $\hat{v}(0) = \frac{\alpha}{q_p v (\alpha + \delta)}$, since $v \geq \hat{v}(0)$, $q_p < q_{p'}$

$$\Rightarrow v \geq \frac{\alpha}{q_p v (\alpha + \delta)} \geq \frac{\alpha}{q_{p'} v (\alpha + \delta)}, \text{ so manager better off}$$

when employee repays privately.

Thus, $\hat{v}(0)=0$ when $q_p < q_{p'}$, so a type 0 manager prefers all violations to be reported privately, and only fixes them if $v \geq \hat{v}(0) = \frac{\alpha}{q_p v (\alpha + \delta)}$.

Now both types of managers want employees to report privately instead of staying silent so both managers will impose maximum penalty for silence and no penalty for private reporting.