PHDBA279 Political Institutions - Problem Set 1

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October 2009

Question 1

Consider the game of "Bribing Voters" as in Dal B (2007). First, in the Laissez-faire voting game consider what happens when play is sequential rather than simultaneous. The simultaneous game, as given in the paper is as follows:

Player 2

		No	Yes
Player 1	No	$0,0,0$	$0,0,0$
	Yes	$0,0,0$	$-\theta,-\theta,-\theta$

Player 3 plays No

Player 2

		No	Yes
Player 1	No	$0,0,0$	$-\theta,-\theta,-\theta$
	Yes	$-\theta,-\theta,-\theta$	$-\theta,-\theta,-\theta$

Player 3 plays Yes

Figure 1: Laissez-faire voting game - Simultaneous Play

From this we can observe that there are five Nash equilibria:
(No, No, No), (Yes, No, No), (No, Yes, No), (No, No, Yes), and (Yes, Yes, Yes).

However, if we eliminate weakly dominated strategies the unique remaining Nash equilibrium is (No, No, No).

The extensive form game tree for the sequential game is shown below. The subscripts on the strategies are included solely for the purpose of exposition. Without loss of generality I assume that player 1 plays first, player two plays second, and player 3 plays third.

Proceeding by backwards induction to find the sub-game perfect Nash equilbria, we observe:

For player three: (No_1, Yes_2), (No_3), (No_5), and (No_7, Yes_8).
For player two: (No_1, Yes_2), and (No_3).
For player one: (No_1, Yes_2).

The key problem is therefor indifference at many nodes, and there is no equivalent extensive form notion of weak dominance to appeal to resolve the conflicts. Note that strategies are complete contingent plans.

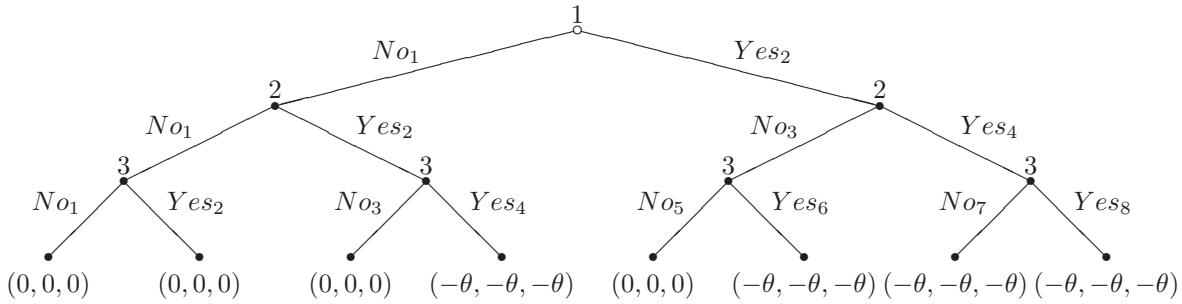

Figure 2: Laissez-faire voting game - Sequential play

The equilibria are therefore:

Player one chooses: (No, Yes).
Player two chooses: $(No, Yes \mid 1\ chooses No)$ and $(No \mid 1\ chooses\ Yes)$
Player three chooses: $(No, Yes \mid 1\ chooses\ No,\ 2\ chooses\ No)$, $(No \mid 1\ chooses\ No,\ 2\ chooses\ Yes)$, $(No \mid 1\ chooses\ Yes,\ 2\ chooses\ No)$ and $(No, Yes \mid 1\ chooses\ Yes,\ 2\ chooses\ Yes)$.

Resulting in the following equilbrium path outcomes:
(No, No, No), (Yes, No, No), (No, Yes, No) and (No, No, Yes).

Thus we can find four, not five, equilbrium path outcome in the extensive form, which has ruled out the (Yes, Yes, Yes) possibility from the strategic form game.

The strategic form game for the "Voting under influence" case is given in the paper as follows:

Player 2

		No	Yes
Player 1	No	$0,0,0$	$0,\epsilon,0$
	Yes	$\epsilon,0,0$	$\epsilon,\epsilon,-\theta$

Player 3 plays No

Player 2

		No	Yes
Player 1	No	$0,0,\epsilon$	$-\theta,\epsilon,\epsilon$
	Yes	$\epsilon,-\theta,\epsilon$	$\epsilon-\theta,\epsilon-\theta,\epsilon-\theta$

Player 3 plays Yes

Figure 3: Voting under influence - Simultaneous Play

The unique Nash equilbrium to this game (for $\epsilon > 0$) is (Yes, Yes, Yes). The extensive form game is as follows:

Again proceeding by backwards induction to find the sub-game perfect Nash equilbria, we observe:

For player three: (Yes_2), (Yes_4), (Yes_6), and (Yes_8).
For player two: (Yes_2), and (Yes_4).
For player one: (Yes_2).

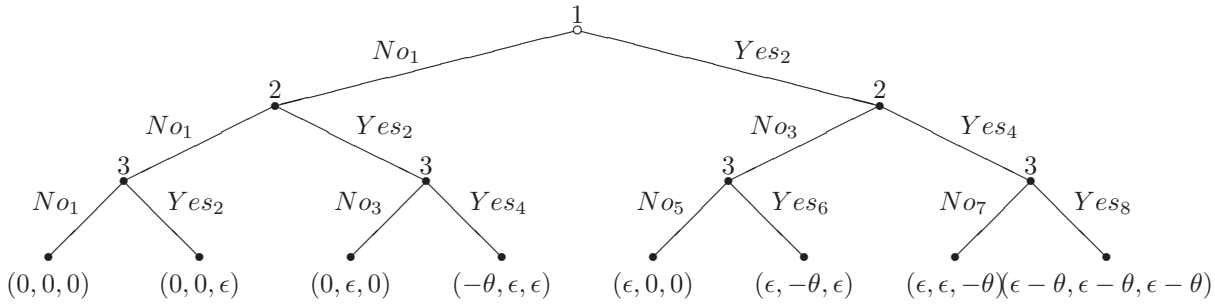

Figure 4: Voting under influence - Sequential play

The equilibria strategies are therefore:

Player one chooses: (Yes).
Player two chooses: $(Yes \mid 1\ chooses\ No)$ and $(Yes \mid 1\ chooses\ Yes)$
Player three chooses: $(Yes \mid 1\ chooses\ No,\ 2\ chooses\ No)$, $(Yes \mid 1\ chooses\ No,\ 2\ chooses\ Yes)$, $(Yes \mid 1\ chooses\ Yes,\ 2\ chooses\ No)$ and $(Yes \mid 1\ chooses\ Yes,\ 2 chooses\ Yes)$.

Resulting in the following in same unique Nash equilibrium as the strategic form game of (Yes, Yes, Yes). Note that the strategies contigent on a previous player choosing No are off the equilibrium path.

Question 2

The Coate and Morris (1995) model is briefly as follows:

Citizens have utility $u_c = y_c + B - T$ where y_c is there initial income, B is the benefit from a project, and T is the tax they face to subsidize transfers.

Special interests have utility $y_s = R + T$, where R is the benefit that they receive from the project and T is the direct transfer.

Good politicians (g) have utility $v_g(u_c - y_c)$, which equals $v_g(B - T)$ by simple substitution. The utility function is increasing in the parameter.

Bad politicians (b) have utility $v_b(u_c - y_c, y_s)$, which equals $v_g(B - T, R + T)$ by simple substitution. The utility function is increasing in both terms and concave.

The paper uses the notion $v_b^*(x, y)$ to denote a bad policians payoff with x indicating the citizen's component without transfers, y indicating the special interests component without transfers, and $*$ indicating that (optimal) transfers are included.

Politicians get a private signal ($\theta \in \{\theta_0, \theta_1\}$) denoting the likely payoff of the project, and their types are private. The game has two periods. In the first period projects and transfers are decided, benefits are realized ($B \in \{B_l, B_h\}$) and an election is held. In the second period politicans can make transfers to special interests. As in the second period transfers are made strictly according to the politician's type, we will ignore this component of the strategy space. Politician's strategies are the pair $(D, T) \in \{P, N\} \times \mathbb{R}_+$, where D, the decision, is either to implement the project (P) or not (N).

The expected benefit of a project to citizens ($\mathbb{E}(B)$) depends on the states of the world and the project decision and is:

$$\mathbb{E}(B) = \begin{cases} \Delta(\theta_0) = \theta_0 B_h + (1-\theta_0)B_l - C & \text{if } P \text{ and } \theta = \theta_0 \\ 0 \quad \text{if } N \text{ and } \theta = \theta_0 \\ \Delta(\theta_1) = \theta_1 B_h + (1-\theta_1)B_l - C & \text{if } P \text{ and } \theta = \theta_1 \\ 0 \quad \text{if } N \text{ and } \theta = \theta_1 \end{cases}$$

The model has four basic assumptions:

1) $\Delta(\theta_1) > 0$ and $\Delta(\theta_0) < -R$, or that the bad (good) project is bad (good) for the citizen.

2)i) $v_b(\Delta(\theta_0), R) > v_b(0,0)$, or that the bad politician would rather implement the bad project than do nothing.

2)ii) $T^*(x,z) = \arg\max V_b(x-T, z+T)$; where T^* is the optimum per period direct-transfer

3)i) $T^*(x,z) \in (0, y_v - C)$, or that the transfer can not bankrupt the citizen (C is a fixed value).

3)ii) $v_b(x,y) + \delta v_b^*(0,0) > v_b^*(x,z)$ or alternatively $v_b(x,y) + \delta v_b(0 - T^*(0,0), 0 + T^*(0,0)) > v_b(x - T^*(x,z), z + T^*(x,z))$, or that the bad politician will forgo a direct transfer in the first period if it will gaurantee him reelection.

4) $v_g(\Delta\theta_1) > v_g(0) + \delta v_g(0)$ and $v_g 0) + \delta v_g(0) > v_g(\Delta\theta_0)$, or that the good politician always behaves efficiently.

The solution concept is perfect bayesian Nash equilibrium, and is characterized by four conditions:

1. Consistent beliefs: The citizens use bayesion updating on their priors (λ_I) and the triple of information received (D, T, B) to form their posteriors (λ_p)
2. The citizen's strategy to releect (or not) the incumbent is optimal given their beliefs, the strategy of the incumbent and the strategy of the challenger

3. The incumbent's strategy is optimal given the beliefs, the strategy of the citizens and the strategy of the challenger
4. The challenger's strategy is optimal.

We do not worry about the 4th characteristic as the challenger is either good or bad and makes transfers according to their type.

Part a

Demonstrate that we cannot have efficient transfers if voters know the politicians type.

In this part we do not need to worry about conditions 1,2, and 4: Beliefs will be consistent as the types are known. That is $\lambda_p = 1$ for a good type and $\lambda_p = 0$ for a bad type. Likewise, the only consistent strategy for the citizen is to reelect a good politican and not to reelect a bad politician (as the worst case is that the challenger is also a bad politician and this leaves the citizen no better off). Again the challenger will make transfers according to their type. Thus the terminating subgame is in equilibrium, beliefs are irrelevant, and we are seeking a simple bayesian Nash equilibrium in the first stage for an overall subgame perfect Nash equilibrium (which is also trivially a perfect Bayesian Nash equilibrium).

We therefore examine the third condition (utility maximization by the incumbent) to determine the equilbrium.

For the good politician:

$$v_g(B-t) = \begin{cases} v_g(\Delta(\theta_0) - T) & \text{if } P \text{ and } \theta = \theta_0 \\ v_g(0 - T) & \text{if } N \text{ and } \theta = \theta_0 \\ v_g(\Delta(\theta_1) - T) & \text{if } P \text{ and } \theta = \theta_1 \\ v_g(0 - T) & \text{if } N \text{ and } \theta = \theta_1 \end{cases}$$

As $v_g(B-t)$ is always decreasing in T, it is clear that a good politician will never make a direct transfer. Furthermore, given assumption 1, the utility maximizing strategy of the good politician is:

$$(D, T) = \begin{cases} (N, 0) & \text{if } \theta = \theta_0 \\ (P, 0) & \text{if } \theta = \theta_1 \end{cases}$$

For the bad politician:

$$v_b(B-T, R+T) = \begin{cases} v_b(\Delta(\theta_0) - T^*(\theta_0, R), R + T^*(\theta_0, R)) & \text{if } P \text{ and } \theta = \theta_0 \\ v_b(0 - T^*(0,0), 0 + T^*(0,0)) & \text{if } N \text{ and } \theta = \theta_0 \\ v_b(\Delta(\theta_1) - T^*(\theta_1, R)), R + T^*(\theta_1, R)) & \text{if } P \text{ and } \theta = \theta_1 \\ v_b(0 - T^*(0,0), 0 + T^*(0,0)) & \text{if } N \text{ and } \theta = \theta_1 \end{cases}$$

As the bad politican is known to be bad and will never get re-elected, he maximizes only his first period utility, and transfers in the second period are irrelevant (this is equivalent to $\delta = 0$).

For $\theta = \theta_0$:

$$v_b(\Delta(\theta_0) - T^*(\theta_0, R), R + T^*(\theta_0, R)) \leq v_b(-R - T^*(\theta_0, R), R + T^*(\theta_0, R))$$

As $\Delta(\theta_0) < -R$ by assumption (1), and as the bad politician's utility function is increasing in both terms.

$$\begin{aligned} & v_b(-R - T^*(\theta_0, R), R + T^*(\theta_0, R)) \\ = & v_b(0 - (R + T^*(\theta_0, R)), 0 + (R + T^*(\theta_0, R))) \\ = & v_b(0 - T'(.,.), 0 + T'(0,0)) \\ \leq & v_b(0 - T^*(0,0), 0 + T * (0,0)) \end{aligned}$$

By assumption (3ii).

For $\theta = \theta_1$:

$$v_b(\Delta(\theta_1) - T^*(\theta_1, R), R + T^*(\theta_0, R)) \geq v_b(-R - T^*(\theta_1, R), R + T^*(\theta_1, R))$$

By assumption (1) $\Delta(\theta_1) > 0$, and it is given that $R > 0$ and the bad politician's utility function is increasing in both terms, but I futher assume that $\Delta(\theta_1) > R > 0$, as otherwise implementing the good project could be inefficient too, which goes against the spirit of the model. Note that this is not necessary to answer the question as given, which is only concerned with the implementation of the project in the bad state of the world.

Thus the utility maximizing strategy of the bad politician is:

$$(D, T) = \begin{cases} (N, T^*(0,0)) & \text{if } \theta = \theta_0 \\ (P, T^*(\theta_1, R)) & \text{if } \theta = \theta_1 \end{cases}$$

Therefore, we cannot have inefficient transfers if the voters know the politician's type.

Part b

Demonstrate that we cannot have inefficient transfers if votes can commit to a particular voting rule.

This could be accomplished with a number of particular voting rules. In general a voting rule where voters do not update their beliefs will yield this result. Examples include voters promising not to kick-out (i.e. to re-elect) an incumbent because he provides a transfer, voters promising to kick-out an incumbent no-matter-what, voters promising to not kick-out an incumbent no-matter-what, and voters promising to randomly kick out an incumbent with some exogenous probability (not based on Bayesian updating).

As a demonstration I take the particular voting rule where voters commit to kicking out the incumbent after the first period, irrespective of what they observe.

Then for the good politician:

$$v_g(B-t) = \begin{cases} v_g(\Delta(\theta_0) - T) & \text{if } P \text{ and } \theta = \theta_0 \\ v_g(0 - T) & \text{if } N \text{ and } \theta = \theta_0 \\ v_g(\Delta(\theta_1) - T) & \text{if } P \text{ and } \theta = \theta_1 \\ v_g(0 - T) & \text{if } N \text{ and } \theta = \theta_1 \end{cases}$$

Again, as $v_g(B-t)$ is always decreasing in T, it is clear that a good politician will never make a direct transfer. Furthermore, given assumption 1, the utility maximizing strategy of the good politician is:

$$(D, T) = \begin{cases} (N, 0) & \text{if } \theta = \theta_0 \\ (P, 0) & \text{if } \theta = \theta_1 \end{cases}$$

Likewise for the bad politician:

$$v_b(B-T, R+T) = \begin{cases} v_b(\Delta(\theta_0) - T^*(\theta_0, R), R + T^*(\theta_0, R)) & \text{if } P \text{ and } \theta = \theta_0 \\ v_b(0 - T^*(0,0), 0 + T^*(0,0)) & \text{if } N \text{ and } \theta = \theta_0 \\ v_b(\Delta(\theta_1) - T^*(\theta_1, R)), R + T^*(\theta_1, R)) & \text{if } P \text{ and } \theta = \theta_1 \\ v_b(0 - T^*(0,0), 0 + T^*(0,0)) & \text{if } N \text{ and } \theta = \theta_1 \end{cases}$$

And the utility maximizing strategy of the bad politician is as above:

$$(D, T) = \begin{cases} (N, T^*(0,0)) & \text{if } \theta = \theta_0 \\ (P, T^*(\theta_1, R)) & \text{if } \theta = \theta_1 \end{cases}$$

Part b

Demonstrate that it is possible to obtain, for suitable parameter values, a different type of equilibrium in which socially valuable projects do not get carried out.

To accomplish this we must first invoke assumption 5 to over-rule assumptions 3ii and 4. Note that the assumptions are rearranged from the format given in the paper; this is done for intuitive clarity.

Assumption 5:

5) i) $v_g(\Delta\theta_1) < v_g(0) + \delta v_g(0)$ the good politican will not implement the project with the good signal

5) ii) $v_b^*(\Delta\theta_1, R) < v_b(0,0) + \delta v_b * (0,0)$ the bad politican will not implement the project with the good signal

We want to "force" the equilibrium:

Good Politican (D,T) =

$$\begin{cases} (N,0) & \text{if } \theta = \theta_0 \\ (N,0) & \text{if } \theta = \theta_1 \end{cases}$$

Bad Politican (D,T) =

$$\begin{cases} (N,0) & \text{if } \theta = \theta_0 \\ (N,0) & \text{if } \theta = \theta_1 \end{cases}$$

In the case of $\theta = \theta_0$, this requires no forcing. We therefore focus on the case where $\theta = \theta_1$. This is forced by the notion that the voters will not re-elect a politician whom they observe playing $(P,0)$. Therefore $\alpha(N,0,0) = \lambda_I$ for both politicians. The utilities of the politicians when $\theta = \theta_1$ are:

$$v_i = \begin{cases} v_g(\Delta(\theta_1)) & \text{if } P \text{ and } i = g \\ v_b^*(\Delta(\theta_1), R) & \text{if } P \text{ and } i = b \\ v_g(0) + \delta G(\lambda_I) v_g(0) & \text{if } n \text{ and } i = g \\ v_b(0,0) + \delta G(\lambda_I) v_b^*(0,0)) & \text{if } N \text{ and } i = b \end{cases}$$

Following the paper, let:

$$G(\lambda^*) = \max\{\frac{v_g(\Delta(\theta_1)) - v_g(0)}{\delta(v_g(0)}; \frac{v_b^*(\Delta(\theta_1), R) - v_b(0,0)}{\delta(v_b^*(0,0)}\}$$

Then:

$$\forall \lambda_I < \lambda^* , \quad v_g(0) + \delta G(\lambda_I) v_g(0) \geq v_g(0) + \delta \frac{v_g(\Delta(\theta_1)) - v_g(0)}{\delta v_g(0)} v_g(0)$$

$$\therefore \forall \lambda_I < \lambda^* , \quad v_g(0) + \delta G(\lambda_I) v_g(0) \geq v_g(\Delta(\theta_1))$$

Therefore the good politician's utility is maximised by choosing $(N, 0)$.

Turning to the bad politician:

$$\forall \lambda_I < \lambda^* \ , \quad v_b(0,0) + \delta G(\lambda_I) v_b^*(0,0) \geq v_b(0,0) + \delta \frac{v_b^*(\Delta(\theta_1), R) - v_b(0,0)}{\delta v_b^*(0,0)} v_b^*(0,0)$$

$$\therefore \forall \lambda_I < \lambda^* \ , \quad v_b(0,0) + \delta G(\lambda_I) v_b^*(0,0) \geq v_b^*(\Delta(\theta_1), R)$$

Therefore the bad politician's utility is also maximised by choosing $(N, 0)$.

Thus, in this equilibrium under assumption 5, we find that socially valuable projects are not implemented by either politician.

Question 3

The policy maker selects a policy $x \in [0, 1]$ and has utility

$$U(x) + c_1 + c_2 = -x^2 + x - c_1 - c_2$$

The interest groups $i = 1, 2$ have utility

$$V_i(x) - f_i - c_i \quad \text{,where } f_i \geq 0$$

and

$$V_i = \begin{cases} -x^2 + 1 & \text{if } i = 1 \\ -x^2 + 2x & \text{if } i = 2 \end{cases}$$

The sequence of the game is:

1. Interest groups simultaneously and non-cooperatively decide whether to organize
2. Organization decisions are known, contribution schedules $c_i(x)$ are made to the policy maker simultaneously and non-cooperatively such that a Truthful Nash equilibrium is played, as in Bernheim and Whinston (1986).
3. The policy maker chooses the policy knowing the contribution schedules offered.

Part a

Solve for the policy selected when no group gets organized.

In stage 1 no group gets organized, so in stage 2 no group can offer a contribution schedule. Therefore in stage 3 the policy maker knowing that no schedule has been offered maximizes his own utility function.

$$\max_x U(x) + c_1 + c_2 \quad \text{s.t.} \quad c_1 = c_2 = 0$$

$$\max_x -x^2 + x \quad \therefore \frac{d}{dx}(-x^2 + x) = -2x + 1 = 0 \quad \therefore x = \frac{1}{2}$$

$$\text{Checking SOC: } \frac{d^2}{dx^2}(-x^2 + x) = -2 < 0 \quad \therefore \text{this is a maximum}$$

Part b

Solve for the policy that each interest group will induce when being the only group that is organised. In each case the policy maker knows the contribution schedule offered and the interest group offering the contribution schedule knows that it is not competing over the policy outcome. The interest group's ideal policy is found by considering $\max_x V(x)$. If a single interest contributes then the problem is a classic (single) principal-agent model. Given that the contributions are transfers and that utility functions are additively seperable for both parties (that is both parties value the contributions seperately from the policy and the same as each other), the solution is one of joint utility maximization.

For interest $i = 1$:

The ideal policy is $\max_x -x^2 + 1$.

Setting the first-order condition equal to zero gives: $2x = 0 \quad \therefore x = 0$.

With interest 1 contributing, the principal chooses:

$$\max_x U(x) + c_1 + V_1(x) - f_1 - c_1 = \max_x U(x) + V_1(x) - f_1$$

$$\therefore \max_x -x^2 + x + -x^2 + 1 - f_1 \quad \therefore \frac{d}{dx}(-2x^2 + x + 1 - f_1) = 0$$

$$\therefore -4x + 1 = 0 \quad \therefore x = \frac{1}{4}$$

For interest $i = 2$:

The ideal policy is $\max_x -x^2 + 2x$.

Setting the first-order condition equal to zero gives: $2x + 2 = 0 \quad \therefore x = 1$.

With interest 2 contributing, the principal chooses:

$$\max_x U(x) + c_2 + V_2(x) - f_2 - c_2 = \max_x U(x) + V_2(x) - f_2$$

$$\therefore \max_x -x^2 + x + -x^2 + 2x - f_2 \quad \therefore \frac{d}{dx}(-2x^2 + 3x - f_2) = 0$$

$$\therefore -4x + 3 = 0 \quad \therefore x = \frac{3}{4}$$

Part c

Now suppose that $f_i = \varepsilon, \varepsilon > 0$. Determine the subgame-perfect equilibrium of the game.

The equilibrium is found in the following manner:

In the subgame (i.e. post-entry) if both principals lobby:

x^* the equilibrium policy is found by maximizing the joint surplus:

$$\max_x u_e(x) + u_1(x) + u_2(x)$$

$$= \max_x -x^2 + x - x^2 + 1 - x^2 + 2x$$

$$= \max_x -3x^2 + 3x + 1$$

$$\frac{d}{dx}(-3x^2 + 3x + 1) = -6x + 3 = 0$$

$$\therefore x = \frac{1}{2}$$

Note that the second order condition indicates that we have found a maximum:

$$\frac{d^2}{dx^2}(-3x^2 + 3x + 1) = -6 \leq 0$$

To calculate the optimum contribution schedule, to specify the equilibrium and determine the utility to each principal, we use the fact that the agent must be indifferent between a compromise policy for the two principals and a policy that favors a single principal. Specifically I assume a linear contribution schedule then solve as follows:

$$c_J^\tau(x, \tau_j) = max\{0, u_j(x) + \tau_j\}$$

Note that this gives local truthtelling around the equilibrium: $c_J^{*\prime}(x^*) = u_J^{*\prime}(x^*)$

$$u_e(x^*) + c_1^*(x^*) + c_2^*(x^*) = u_e(x_1) + c_1^*(x_1)$$

$$u_e(x^*) + u_1^*(x^*) + u_2^*(x^*) + \tau_1 + \tau_2 = u_e(x_1) + u_1^*(x_1) + \tau_1$$

$$u_e(x^*) + u_1^*(x^*) + u_2^*(x^*) + \tau_2 = u_e(x_1) + u_1^*(x_1)$$

$$\tau_2 = u_e(x_1) + u_1^*(x_1) - (u_e(x^*) + u_1^*(x^*) + u_2^*(x^*))$$

Likewise:

$$\tau_1 = u_e(x_2) + u_2^*(x_2) - (u_e(x^*) + u_1^*(x^*) + u_2^*(x^*))$$

Substituting in this gives:

$$\tau_1 = -(\frac{3}{4})^2 + \frac{3}{4} + -(\frac{3}{4})^2 + 2\frac{3}{4} - (-(\frac{1}{2})^2 + \frac{1}{2} - (\frac{1}{2})^2 + 1 + -(\frac{1}{2})^2 + 2\frac{1}{2})$$

$$= -2\frac{9}{16} + 3\frac{1}{4} - (-3(\frac{1}{2})^2 + 3\frac{1}{2} + 1) = \frac{9}{8} - \frac{14}{8}$$

$$\therefore \tau_1 = -\frac{5}{8}$$

$$\tau_2 = -(\frac{1}{4})^2 + \frac{1}{4} + -(\frac{1}{4})^2 + 1 - (-(\frac{1}{2})^2 + \frac{1}{2} + -(\frac{1}{2})^2 + 1 + -(\frac{1}{2})^2 + 2\frac{1}{2})$$

$$= -2\frac{1}{16} + \frac{1}{4} + 1 - (-3(\frac{1}{2})^2 + 3\frac{1}{2} + 1) = \frac{9}{8} - \frac{14}{8}$$

$$\therefore \tau_2 = -\frac{5}{8}$$

Likewise I solve for the contribution schedules when only one principal contributes. As part b only asked for the optimum policy, this was not required above.

$$u_e(x^1) + c_1^*(x^1) = u_e(x_A)$$

$$\rho_1 = u_e(x_A) - ((u_e(x^1) + u_1^*(x^1))$$

$$\rho_1 = -(\frac{1}{2})^2 + \frac{1}{2} - (-(\frac{1}{4})^2 + \frac{1}{4} + -(\frac{1}{4})^2 + 1)$$

$$\rho_1 = -\frac{2}{8}) + \frac{4}{8} - (-(\frac{1}{8}) + \frac{2}{8} + 1)$$

$$\rho_1 = -\frac{7}{8}$$

Likewise:

$$u_e(x^*2) + c_2^*(x^*2) = u_e(x_A)$$

$$\rho_1 = u_e(x_A) - ((u_e(x^2) + u_2^*(x^2))$$

$$\rho_2 = -(\frac{1}{2})^2 + \frac{1}{2} - (-(\frac{3}{4})^2 + \frac{3}{4} + -(\frac{3}{4})^2 + 2\frac{3}{4})$$

$$\rho_2 = -(\frac{2}{8}) + \frac{4}{8} - (-(\frac{9}{8}) + \frac{18}{8})$$

$$\rho_2 = -\frac{7}{8}$$

Therefore, looking across all of the cases we see that the first stage (and noting that $V_i = u_i(.) - c(.) = \rho_i$ in the single principal case and $V_i = u_i(.) - c(.) = \tau_i$ in the two principal case.

$$V_1 = \begin{cases} u_1(x_1) - \epsilon = \frac{7}{8} - \epsilon \quad \text{if only 1 enters} \\ u_1(x_2) = -x_2^2 + 1 = -(\frac{3}{4})^2 + 1 = \frac{7}{16} \quad \text{if only 2 enters} \\ u_1(x_A) = -x_A^2 + 1 = -(\frac{1}{2})^2 + 1 = \frac{3}{4} \quad \text{if neither enter} \\ u_1(x^*) - \epsilon = \frac{5}{8} - \epsilon \quad \text{if both enter} \end{cases}$$

$$V_2 = \begin{cases} u_2(x_1) = -x_1^2 + 2x_1 = -(\frac{1}{4})^2 + 2\frac{1}{4} = \frac{7}{16} \quad \text{if only 1 enters} \\ u_2(x_2) - \epsilon = \frac{7}{8} - \epsilon \quad \text{if only 2 enters} \\ u_2(x_A) = -x_A^2 + 2x_1 A = -(\frac{1}{2})^2 + 2\frac{1}{2} = \frac{3}{4} \quad \text{if neither enter} \\ u_2(x^*) - \epsilon = \frac{5}{8} - \epsilon \quad \text{if both enter} \end{cases}$$

This results in the stage one game:

		Player 2	
		$Enter$	$Don't Enter$
Player 1	$Enter$	$\frac{5}{8} - \epsilon, \frac{5}{8} - \epsilon$	$\frac{7}{8} - \epsilon, \frac{7}{16}$
	$Don't Enter$	$\frac{7}{16}, \frac{7}{8} - \epsilon$	$\frac{3}{4}, \frac{3}{4}$

Figure 5: Stage 1 - Entry decision

Or, for ease of viewing, multiplying all payoffs by 16 :

		Player 2	
		$Enter$	$Don't Enter$
Player 1	$Enter$	$10 - \epsilon, 10 - \epsilon$	$14 - \epsilon, 7$
	$Don't Enter$	$7, 14 - \epsilon$	$12, 12$

Figure 6: Stage 1 - Entry decision

From the above figure it should be evident that there are five cases:

$$\epsilon \begin{cases} \in (0, \frac{2}{16}) \quad \text{then there is a prisoner's dilemma. The single SPNE is } (Enter, Enter) \\ = \frac{2}{16} \quad \text{then there are two pure strategy SPNE } (Enter, Don't), (Don't, Enter) \\ \in (\frac{2}{16}, \frac{3}{16}) \quad \text{then there are two pure strategy SPNE } (Enter, Don't), (Don't, Enter) \\ = \frac{3}{16} \quad \text{then there are two pure strategy SPNE } (Enter, Don't), (Don't, Enter) \\ \in (\frac{3}{16}, \infty) \quad \text{then the single SPNE } (Don't, Don't) \end{cases}$$

Note that when ever there is more than one pure strategy SPNE, then mixed strategies are also possible. Further more in the case where $\epsilon = \frac{2}{16}$ the weakly

dominant (and so most likely) strategy is $(Enter, Enter)$, whereas in the case where $\epsilon = \frac{3}{16}$ the weakly dominant (and so most likely) strategy is $(Don't, Don't)$. In addition, the full specification of the equilibria includes the contribution schedules and policy outcomes found above.

Comment on the properties of this equilibrium, especially in relation to the efficiency properties of the equilibrium in Grossman and Helpman (1994):

The contributions themselves are costless transfers and lead to no inefficiency. The fixed cost to organize, however, represents an efficiency loss. If the fixed cost is at or below $\frac{2}{16}$ then this inefficiency is realised for both parties leading to a total loss of $\frac{1}{4}$. Between $\frac{2}{16}$ and $\frac{3}{16}$ the fixed cost is an efficiency loss for only one interest, and above the $\frac{3}{16}$ the fixed cost is never spent and so never lost.

Part d

Two options are presented to curb business influence on policy.

Option 1) Prohibit (enforceably) one interest from organizing. Option 2) Raise the fixed cost (ϵ) for both groups.

Option 2 is more appropriate to eliminate waste and inefficiency providing the fixed cost is raised above $\frac{3}{16}$ - this will prevent either interest from incurring the (wasteful) fixed cost and deter both groups from lobbying.

However, if the fixed cost is raised to less than $\frac{2}{16}$, this will not deter lobbying by either group, but will increase the waste from ϵ to $\epsilon\prime \geq \epsilon$. In such a case option 1 is more appropriate.

Likewise if $\epsilon\prime \in (\frac{2}{16}, \frac{3}{16})$ this will have the effect of detering one interests lobbying but not the other in equilibrium. Note that as the interests may adopt a mixed strategy there may be times when both enter and times when neither enter. However, this may be worse than the prohibition on a single interest from entering (from a waste perspective), as now $\epsilon\prime \geq \epsilon$ and more waste is potentially incurred. From a "fairness" perspective though, it could be argued that this is fairer that the prohibition. Ignoring "fairness", option 2 is more appropriate here.

Overall, given that the game is common knowledge to all of the players, one of who is the government (the executive), it should be possible to force the fixed cost to be greater than $\frac{3}{16}$ and this makes option 1 the best option.

Part e

How would you use this model to study the causes and consequences of lobbying in the relation to the pharma industry?

The majority of lobbying the pharma industry appears to be industy vs. special interests, rather than firm vs. firm. As such one would expect that it is comparatively easy for the industy to organize (the fixed cost is low relative to the gain for even a single firm) and comparatively difficult for the special interests to organize. We expect, however, that on big issues (e.g. stem cell research) that interest groups will organise too. The model then predicts an equilibrium outcome of no policy change from the status-quo (actually from the executive's preferences), but meaningful transfers to the politician. We certainly do observe extensive lobbying by both sides and it is often unclear who benefits from individual policies, which provides prima facie support to the model.

What limitations would you find in applying this model?

Many policies are binary in nature. Specifically, stem cell research for example, is generally viewed as either allowed or disallowed, rather than a continuous spectrum of allowance. The model requires a continuous policy space and continuous preferences by all parties.

Furthermore outcomes are often highly uncertain and contingent on policy in a non-linear fashion. For example, it may be hard for a pharma firm to quantify the gain or loss that it will get from a policy, and it may be able to 'invent-around' certain outcomes.

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