MBA201A - Option Value Handout

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A simple example

An option is a right to do take some action in the future (e.g. to buy or sell some quantity at a preset price). There are four characteristics to bear in mind:

1. You buy the option ahead of the time that is it needed
2. Options have a cost (which may be zero) attached to them
3. You don't have to exercise an option - you only do so if it is worthwhile
4. An option must affect the payoffs in one or more states of the world to be relevant

Consider the following decision tree:

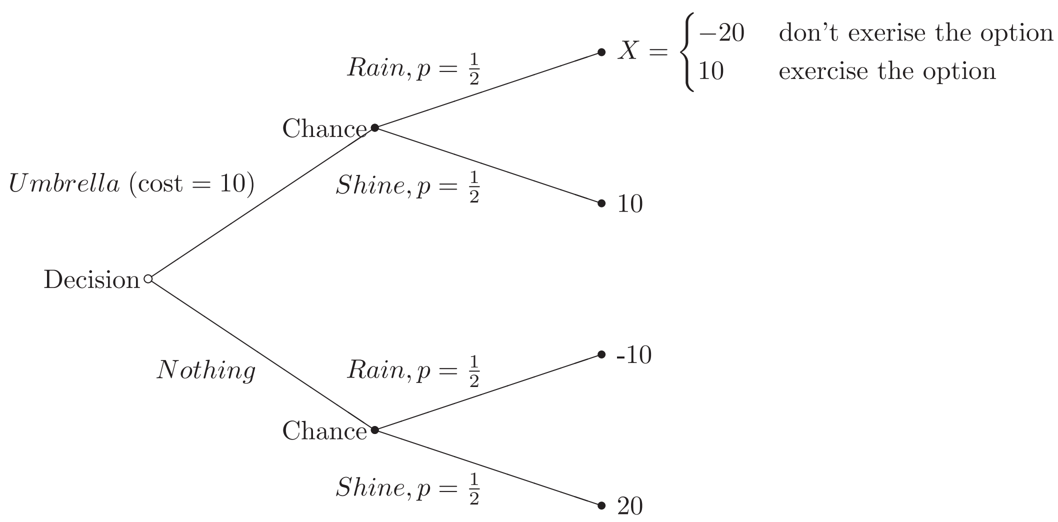

Figure 1: An option to take an umbrella

In figure 1 the payoffs on the terminal nodes include the cost of the option.

In their simplest form, options are involve two decisions: the purchase of the option, and the exercise of the option. Recall that I said you don't have to exercise an option. However, if we were to end up in the rain state, clearly we would want to exercise the option for a payoff of 10! In most cases you don't need to model the exercise of an option - just write up the payoffs appropriately. If you want to model the exercise decision, it should always involve strict dominance.

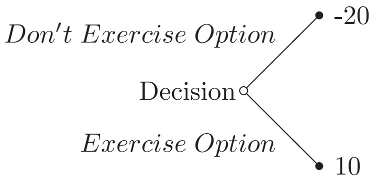

Figure 2: The exercise decision for the umbrella option

In figure 2 there is no chance node. When you apply backwards induction, you simply choose the highest payoff.

You should always model the decision to purchase an option though, even if it has zero cost, because you can use the purchase decision branches to calculate the option value.

The option value is the difference in the expected values when you purchase the option and when you don't. We denote a difference with a Δ (capital delta). Therefore:

$$OptionValue(Action) = \Delta\mathbb{E}(Action)$$

$$= \mathbb{E}(Action\ With\ Option) - \mathbb{E}(Action\ Without\ Option)$$

$$\therefore OptionValue(Umbrella) = \Delta\mathbb{E}(Umbrella) = \mathbb{E}(Umbrella) - \mathbb{E}(Nothing)$$

where

$$\mathbb{E}(Umbrella) = \frac{1}{2} \times 10 + \frac{1}{2} \times 10 = 10$$

$$\mathbb{E}(Nothing) = \frac{1}{2} \times -10 + \frac{1}{2} \times 20 = 5$$

$$\therefore OptionValue(Umbrella) = \Delta\mathbb{E}(Umbrella) = 10 - 5 = 5$$

How much would you be willing to pay for this option? The answer is the entire option value. That is, you would be willing to pay the cost of the option as given, plus another 5 units of utility.

Net and Gross Option Value

Below figure 1 there was a note saying that the payoffs include all costs. Thus the payoffs for rain when you exercise option are 10 including the cost of the option, and the payoff when you don't exercise the option is -20, instead of -10. (Again, of course, you are going to exercise the option.) However, it is very important to note that you are calculating a Net Option Value, and not a Gross Option Value in this case. With a Net Option Value the costs are included, so to decide whether the option is worthwhile you compare it to 0:

$$OptionValue_{Net} > 0 \quad \implies \text{The option has positive value}$$

We could also use a Gross Option Value. The figure below redraws figure 1 to use gross payoffs.

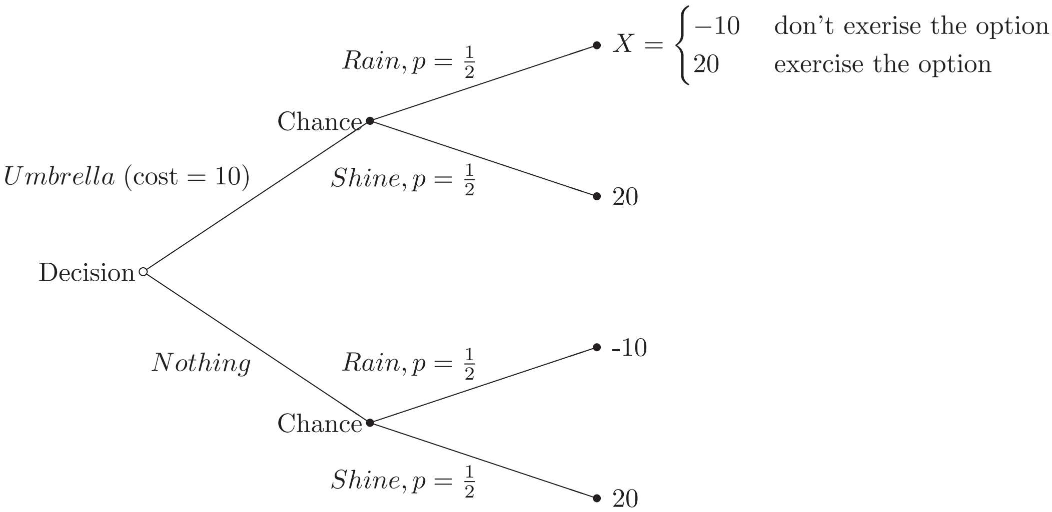

Figure 3: An option to take an umbrella, with gross payoffs

Now we calculate the gross option value as before, by calculating the difference in expectations:

$$OptionValue(Action)_{Gross} = \Delta\mathbb{E}(Action)_{Gross}$$

$$\Delta\mathbb{E}(Action)_{Gross} = \mathbb{E}(Action\ With\ Option)_{Gross} - \mathbb{E}(Action\ Without\ Option)_{Gross}$$

$$\begin{aligned} \therefore OptionValue(Umbrella)_{Gross} &= \Delta\mathbb{E}(Umbrella)_{Gross} \\ &= \mathbb{E}(Umbrella)_{Gross} - \mathbb{E}(Nothing)_{Gross} \end{aligned}$$

where

$$\mathbb{E}(Umbrella)_{Gross} = \frac{1}{2} \times 20 + \frac{1}{2} \times 20 = 20$$

$$\mathbb{E}(Nothing)_{Gross} = \frac{1}{2} \times -10 + \frac{1}{2} \times 20 = 5$$

$$\therefore OptionValue(Umbrella)_{Gross} = \Delta\mathbb{E}(Umbrella)_{Gross} = 20 - 5 = 15$$

But for Gross Option Values we need to make a comparison to the cost of the option:

$$OptionValue_{Gross} > cost \quad \implies \text{The option has positive value}$$

With Gross Option Values you would be willing to pay the entire Gross Option Value for the option. Think about it: If you paid 14 for the option above, you'd still be 1 unit of utility better off than if you didn't have the option. Likewise if you paid 14.99 you would be 0.01 units better off. You would be indifferent between paying 15 units and not having the option at all - so 15 units (the Gross Option Value) is your maximum willingness to pay.

To calculate the Net Option Value from the Gross Option Value:

$$OptionValue_{Net} = OptionValue_{Gross} - cost$$

.

$$\therefore OptionValue_{Net} = 15 - 10 = 5$$

.

I recommend that you work with Net Option Values, and that you also record the cost on the decision branch. However, when working with Net Option Values and asked how much you would be willing to pay for an option, remember that you would be willing to pay the existing cost of the option plus the Net Option Value.

Quick Trick

How else could we have done this? Notice that in the Gross Option Value example only the rain state has a diffence of $20 - (-10) = 30$ in the payoffs when you exercise the option, and this occurs with probability $\frac{1}{2}$.

$$30 \times \frac{1}{2} = 15 = OptionValue \ (Gross)$$

This shortcut works only because the other state (shine) has the same payoffs regardless of the option - it is a mathematical trick (look at the net payoffs and convince yourself that it wouldn't work to ignore the shine state there, and understand that it has nothing to do with net or gross)!

Is it this simple?

Yes, it is this simple! Suppose you have a simple decision tree:

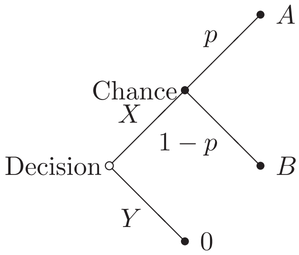

Figure 4: A simple tree

And now you are asked given the ability to buy some option before hand.

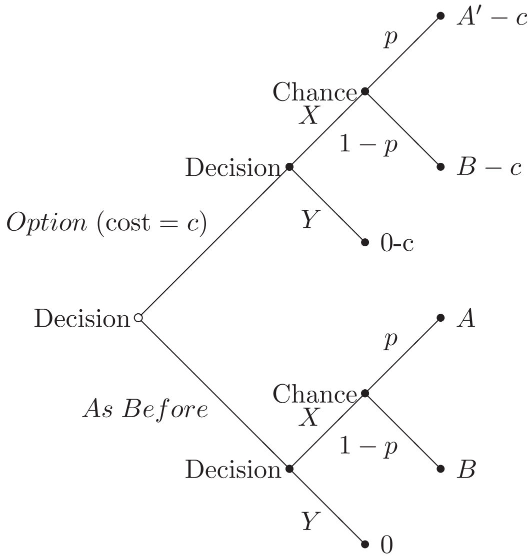

Figure 5: Option over a simple tree

You would do your expected value calculations (and make any necessary decisions) back to your two option action branches (i.e. *Option* and *As Before*), and then calculate your option value.

Just remember:

- To add the cost of the option to the option branch
- To substract the cost of the option from the payoffs, so that you are working in Net Option Values

- To change any payoff where you exercise the option (here I marked one with a prime - A')

Of course, you might consider it bad practise to have two layers of decisions next to each other. This is not a problem - just collapse them as in figure 6, and again calculate your expected values and make any necessary decisions back to the option decision node. Here the option value is the difference in the expected value between the decision that you would make with the option and the decision you would make without the option.

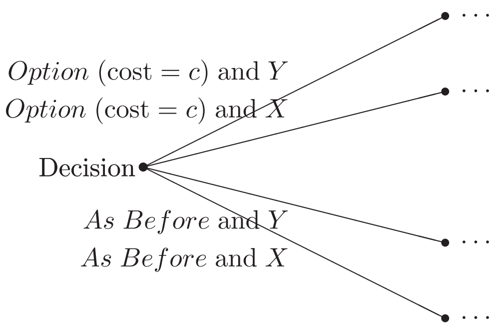

Figure 6: Option over a collapsed tree

Information Value

Calculating information value is the much same as calculating option value, with two important differences. Now, instead of decision node at the start of the tree, there will be a “nature” node, as in the diagram below, where in one state nature reveals the information and in the other the information is not revealed as before. For the state with the information revealed, you should reverse the order of the chance and decision nodes. That is the chance node that will have its information revealed should now come before the decision in question.

In the state where the information is revealed, you ‘prune’ your tree, removing all branches that are dominated. In the diagram above, if $A > 0 > B$ then the dominated payoffs are crossed out. As the decision now occurs after the chance node, you do not need to calculate an expectation for these decision nodes - dominance can always be used to determine the best action. Then you back up the expected value over the chance node using the probabilities as weights (in the example above this is $p \cdot (A - c) + (1 - p) \cdot (0 - c)$) and calculate the difference in expected values for the two information branches (you will only ever see two) plus any cost (assuming you are working with net payoffs and have subtracted

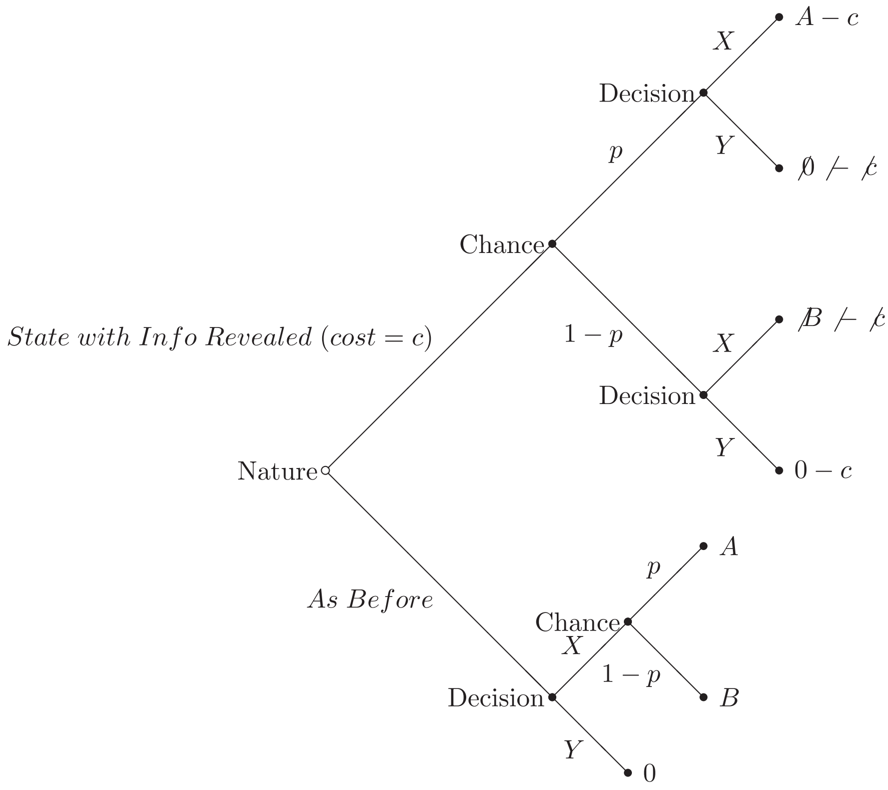

Figure 7: Solving for information value

the cost from them already). If there is no cost then net payoffs are the same as gross payoffs. For gross payoffs you just calculate the difference in expected values for the two information branches. Either way, this is the information value.

$$InformationValue = \Delta\mathbb{E}_{Gross}$$

$$= \mathbb{E}(Decision|Information\ Revealed) - \mathbb{E}(Decision|Information\ Not\ Revealed)$$

If the information is costly, then add it to the diagram in the same way as the option cost was added, and again I suggest you work with Net Payoffs. The value of the information is then the same as the value of an option - your total willingness to pay for the information is either the Gross Information Value, or the Net Information Value plus the pre-existing information cost.

$$InformationValue = \Delta\mathbb{E}_{Net} + cost$$

In the example above this is:

$$\mathbb{E}(Decision|Information\ Revealed) = p \cdot (A - c) + (1 - p) \cdot (0 - c)$$

$$\mathbb{E}(Decision|Information\ Not\ Revealed) = \max\{p \cdot A + (1-p) \cdot B\ ,\ 0\}$$

$$InformationValue = \Delta\mathbb{E}_{Net} + c$$

$$= (p \cdot (A-c) + (1-p) \cdot (0-c)) - (\max\{p \cdot A + (1-p) \cdot B\ ,\ 0\}) + c$$

In the context of the umbrella problem in figure 1, if nature reveals the state to be 'rain', then in the tree that extends from the *State with Information Revealed* branch you should first move the chance node forward of the decision node, then prune away all of the dominated payoffs.

First, consider the umbrella example without any information (and without an option - so it is just a decision to take an umbrella or not) but with slightly changed payoffs.

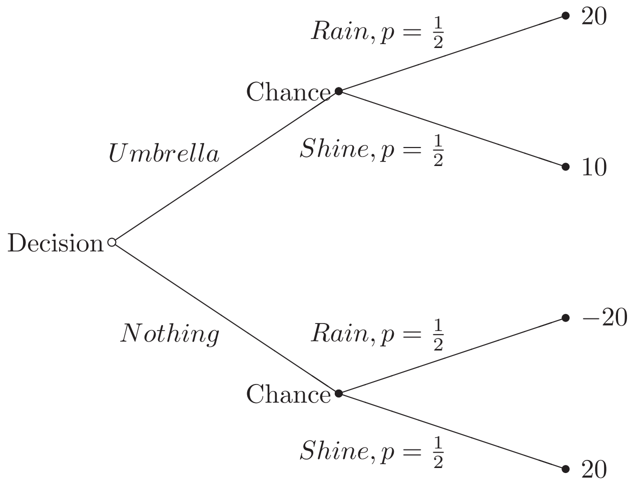

Figure 8: A decision to take an umbrella

The expected value of the *Umbrella* decision is 15 and is higher than the expected value of the nothing decision. Therefore you will take the umbrella (for 15 units of utility). Now we draw the information value tree.

The expected value at the chance node (of Rain or Shine) is 20 (see figure 9 below). Therefore:

$$InformationValue = \Delta\mathbb{E} = \mathbb{E}(With\ Info) - \mathbb{E}(As\ Before) = (20-15) = 5$$

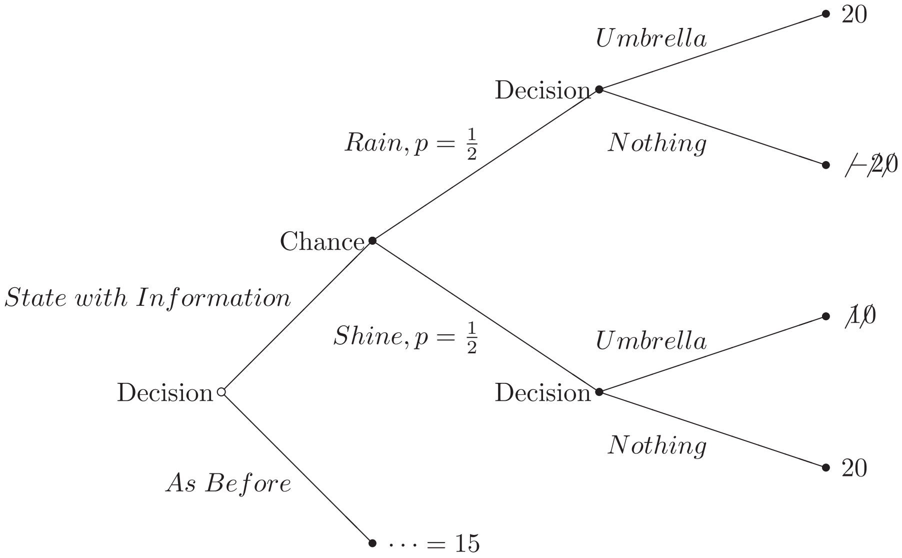

Figure 9: Umbrella Information Value example