MBA201A: Advanced Pricing Handout

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October 2010

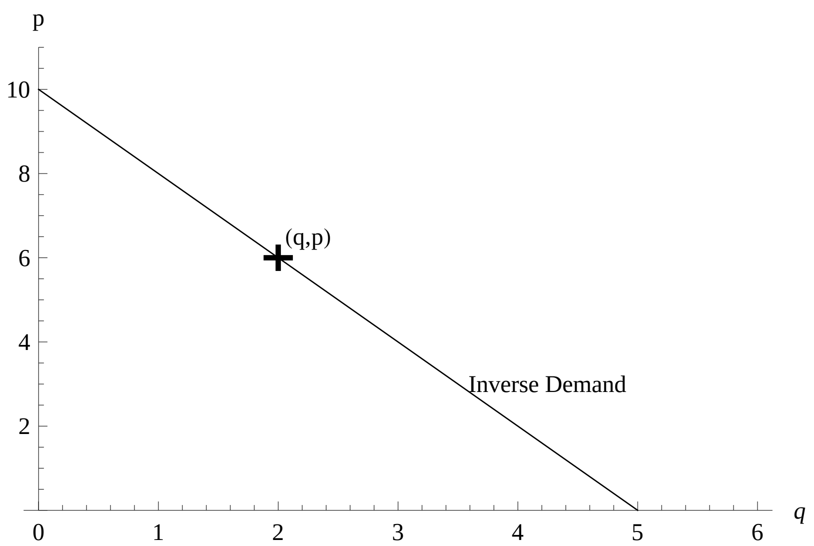
Introduction

This handout covers:

- **1st Degree Price Discrimination -** Charging customers their full willingness to pay
- **Bundling -** Selling two or more products together in a bundle.
- **2nd Degree Price Discrimination -** Using Packaging/Versioning to reveal which unobservable type a customer belongs to
- **3rd Degree Price Discrimination -** Using observable characteristics to discriminate between groups of customers
- **Two Part Tariffs -** Charging customers an entry fee as well as a price per unit
- **Competitive Markets -** Short and long run equilibriums in competitive markets

Every price-quantity[1] pair (q,p) that exists in the economy must lie on the (inverse) demand curve, regardless of the whether we are using monopoly pricing, price discrimination, or competitive pricing.

[1]By convention we talk about price-quantity pairs and not quantity-price pairs, though the latter would be more correct. We plot quantity on the x-axis and price on the y-axis, and so we write (q,p) as we would write (x,y).



Competition

Regardless of whether we are looking at short or long run equilibriums, **in competitive markets it is always true that price equals marginal cost**:

$$p = mc$$

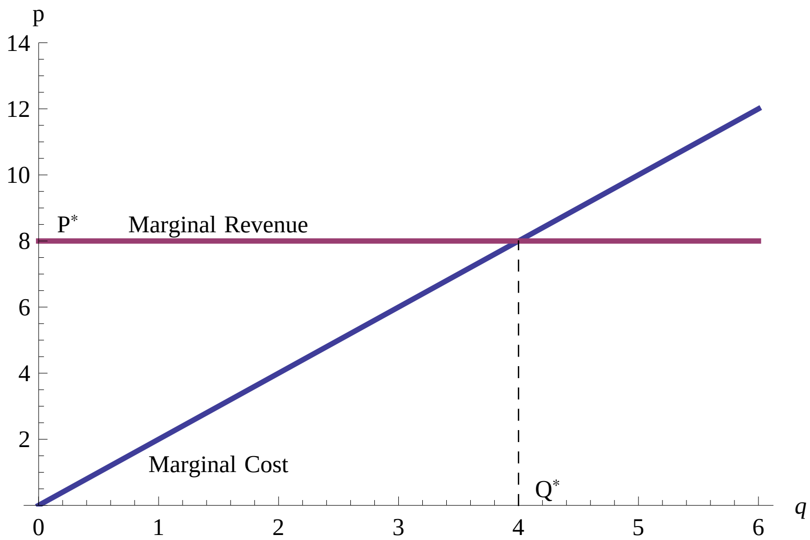
This is in contrast to the monopoly world where marginal revenue equals marginal cost ($mr = mc$) because profits are being maximized ($\pi' = 0 \implies mr = mc$).

In a competitive world there are many firms selling the same product (i.e. it is a commodity) to a large number of consumers, and there are no barriers to entry. (Although as we will see, sometimes 2 firms will do!).

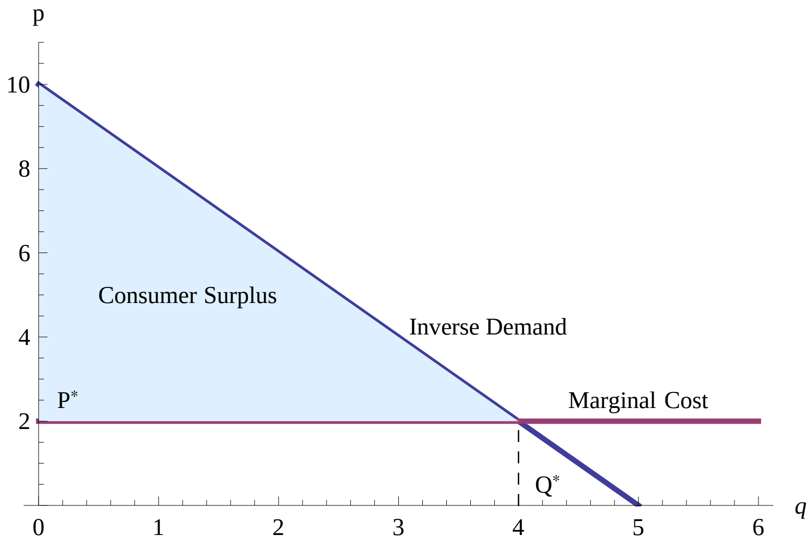
There are two ways of getting to the equation $p = mc$:

1. Consider all the firms to be price takers that choose how much quantity to produce (and whether to produce at all). As the firms are price takers they can't affect the price, and it is the same regardless of the quantity that they sell. The price is therefore the marginal revenue of each unit, and it is a flat line. See the figure below.

2. Consider two firms producing the same good and competing on price. This leads to the Bertrand Trap, discussed in the Game Theory Handout, where if one were to price above marginal cost the other would undercut them and take the entire market. Pricing below marginal cost is clearly not an option as it would involve taking a loss on every sale, so the firms must price at marginal cost.



We will return to competition later, but for now we should note one important property of competition - it leads to efficient allocation of goods. Every customer with a willingness to pay for a good above the marginal cost does indeed get the good, or alternatively (and equivalently), a customer buys the efficient quantity of a good. The area above the price and below the demand curve is the consumer surplus.



1st Degree Price Discrimination

First degree price discrimination is simply charging each and every consumer thier entire willingness to pay. In practice this is essentially impossible (we could perhaps auction every unit of every good in a second price auction) unless we are both able to see each persons willingness to pay and able to actually charge it. Arbitrage by consumers may prevent the latter, and we often assume that goods are non-transferable, such as goods for immediate consumption, so that arbitrage is not possible.

There are two versions of 1st degree price discrimination - one for when demand is discrete and another for when demand is continuous. With continous demand we need to assume that consumers are homogenous - that is they all have the same demand functions. Then we extract each consumer's surplus.

$$CS = \text{Area under demand and above price} = \frac{1}{2}\times\text{base}\times\text{height} = \frac{(Intercept - p) \times q}{2}$$

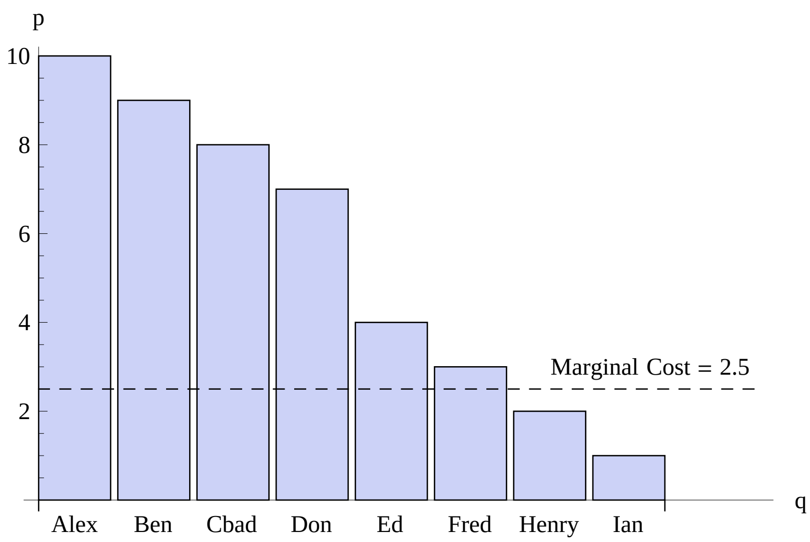
In the case of discrete demand, we assume that each consumer has a different willingness to pay. We can then put the consumers in order of decreasing willingness to pay on a graph. We can charge a consumer up to their willingness to pay. So if we were using simple pricing (i.e. charge a single price for everyone) and charged p, we would sell to all customers whose WTP was greater than or equal to p, and every customer with a greater willingness to pay than the price would get some consumer surplus.

In first degree price discrimination we charge each customer their exact willingness to pay.

Supposing that we had eight consumers with the following willingesses to pay:

Name	**WTP**
Alex	10
Ben	9
Chad	8
Don	7
Ed	4
Fred	3
Henry	2
Iain	1

Plotting these values on a graph gives:



Thus under simple pricing if we charged 9, then Alex and Ben would purchase the good and only Alex would get some consumer surplus (1 unit). However, under first degree price discrimination we would charge each consumer their willingness to pay, providing it exceeds our marginal cost of production, so we would charge Alex 10, Ben 9, Chad 8, and so forth.

Bundling

In the example above, with the eight consumers (Alex through Ian), what was the optimum simple price? If the price is 10, we sell one unit (to Alex), whereas if the price is 9 we sell two units (one to Alex and one to Ben), and so forth. If our marginal cost is 2.5 then we could tabulate the revenues and profit contibutions (i.e. profit excluding fixed cost deductions) as follows:

Price	**Units Sold**	**Revenue**	**Profit Contrib.**
10	1	10	7.5
9	2	18	13
8	3	24	16.5
7	4	28	18
4	5	20	7.5
3	6	18	3

Clearly we do not want to be selling below marginal cost, so we will never sell to Henry or Ian. As it turns out we do not want to sell to Ed or Fred either. In the table above the profit contribution is maximized when we sell 4 units at a price of 7. This is because selling to Ed has an opportunity cost (of not selling at the higher price you could sell to Don at) that is greater than the benefit of the revenue that Ed's purchase bring in.

In practice you will solve these problems by building a table of Revenue and Profit Contribution, as above. However, to make this idea concrete we will now solve for the explicit tradeoff of selling to the next person. Let the price that you would sell to person N at be p and then the price that you would have to charge to get person $N+1$ to buy is $p - \Delta$.

We will sell to person $N+1$ if the profit contribution from adding that last person is greater than the profit contribution without them, that is if:

$$(N+1) \cdot ((p - \Delta) - c) > N \cdot (p - c)$$
$$\therefore (p - c) > (N+1) \cdot \Delta$$

In the case above, we were charging the 4th ($N = 4$) person, Don, $p = 7$, and then tried to add the 5th ($N + 1 = 5$) person, Ed, which involved dropping the price by $\Delta = (7 - 4) = 3$. With our marginal cost of $c = 2.5$ this gives us:

$$(7 - 2.5) \not> 5 \cdot 3 \quad \text{because} \quad 4.5 \not> 15$$

So in this example we do not want to sell to Ed, because the opportunity cost is too high.

With more than one good and only simple pricing we repeat this exercise for each good seperately. However, if the correlation between different customers' (or types of customers) willingness to pay is not 1 (i.e. if the customers or groups of customers value the two goods relatively differently), then you will be able to do better (make higher profits) with a bundling scheme.

There are then two types of bundling:

- **Pure Bundling** - when only the bundle is sold to any consumer type that buys
- **Mixed Bundling** - when the bundle is sold to one or more consumer types, and at least one single good is sold to at least one consumer type who doesn't buy the bundle.

Suppose we have two groups (A and B), who have willingnesses to pay for two goods (1 and 2) greater than marginal cost (say $c = 1$) as follows: $WTP_A1 = 10$, $WTP_A2 = 20$, $WTP_B1 = 20$, and $WTP_B2 = 10$. Each groups willingness to pay for the bundle is simply the sum of their willingnesses to pay for each good seperately. The table below shows this information, as well as profit contributions in brackets (note that in the bundle there are two goods).

Group	**WTP for** 1	**WTP for** 2	**WTP for Bundle**
A	10 (9)	20 (19)	30 (28)
B	20 (19)	10 (9)	30 (28)

Then for each good it is straight forward to calculate the revenue and profit contribution that we would make at each price we could charge (note that we are never going to charge a group less their willingness to pay, unless we are pricing to entice another group with a lower WTP to purchase, as this would be throwing away revenue):

Good	**Price**	**Quantity**	**Revenue**	**Profit Contrib.**
1	10	2	20	18
1	20	1	20	19
2	10	2	20	18
2	20	1	20	19
Bundle	30	2	60	56

In this example, if we are constrained to simple pricing then the best we can do is to sell each good at 20 and to make a single sale (to give us a profit contribution of 19). However, if we bundle the two goods together and sell the bundle at 30, then we sell a bundle to both groups and nearly triple our profit contribution!

Mixed bundling means that some customers buy the bundle and others purchase single items. It may be prohibitive to calculate all the possible combinations of sales (i.e. sell good 1 to group A, good 2 to group B, the bundle to group C, and so forth). In this case you should focus on selling discrete goods to any groups that are outliers in terms of their valuations. When you have a candidate mixed bundling scheme you must check that no group would want to defect from purchasing thier intended 'good', whether that be the bundle or one or more single items, to another offered 'good'.

2nd Degree Price Discrimination

This year, the mathematical tools required to solve second degree price discrimination problems were not taught. However, the concept of second degree price discrimination certainly was covered and you should know that:

- 2nd degree price discrimination is used when types are **unobservable**
- The optimal pricing scheme will use **packaging/versioning** to get the types to **reveal themselves**
- The solution has **efficiency at the top** (for the high type) and **no rents at the bottom** (for the low type)
- The solution gives **information rents** to the high type to reward them for revealing themselves and to prevent them from pretending to be low types

In addition, you may recall from the problem sets that you should check your answer against a simple pricing solution. While (any type of) price discrimination will, **for sure**, do (weakly) **better than simple pricing** when all groups are served by simple pricing, if the simple pricing optimum solution is not serve a group of consumers, then this gaurantee is no longer in place!

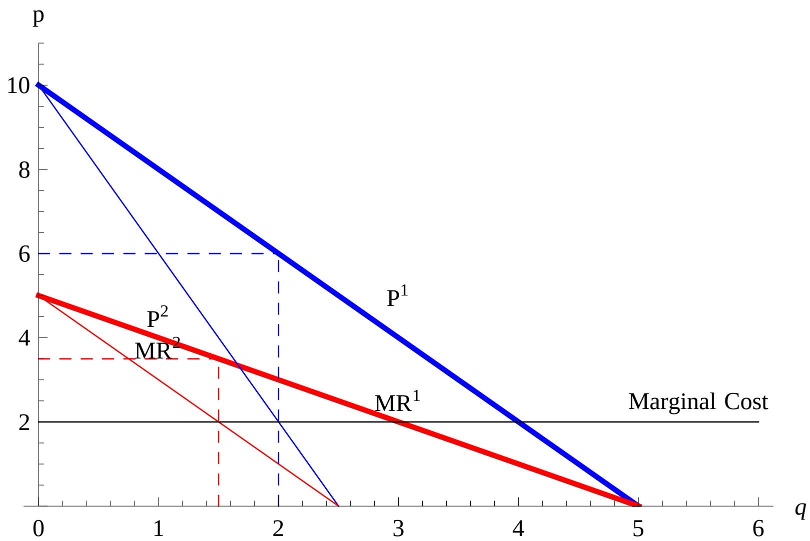
The canonical example of second degree price discrimination is first class vs. economy class airline tickets.

3rd Degree Price Discrimination

Third degree price discrimination is used when the differences between groups are observable. The canonical example of second degree price discrimination is student or OAP (old age pensioner) discounts; in both cases an I.D. card can be produced to confirm eligibility for a certain price.

There are two approaches to third degree price discrimination. The first **applies only when marginal cost is constant**, and is very easy: Solve the monopoly price and quantity for each group seperately, and do not double count fixed costs.

That is each group gets their own price and quantity and so their own revenue. Total costs are then calculated either using the total quantity, or making sure not to double count fixed costs.



In equations we solve $MR = MC$ once for each demand curve to get a set of price-quantity pairs. With two demand curves, we will have two price-quantity pairs:

$$MR_1 = MC \quad \Longrightarrow (q_1, p_1)$$

$$MR_2 = MC \quad \implies (q_2, p_2)$$
$$\pi = R - C = R_1 + R_2 - (FC + MC(q_{total})) \quad \text{where } q_{total} = q_1 + q_2$$
$$\therefore \pi = p_1 \cdot q_1 + p_2 \cdot q_2 - (FC + MC(q_1) + MC(q_2))$$

However, we have to be **very careful if marginal cost is not constant**. If marginal cost is increasing (or decreasing) then the last line of that equation does not hold as $MC(q_{total}) \neq MC(q_1) + MC(q_2)$. This leads us to our second and general approach.

In class you were taught that to solve this problem you should set:

$$MR_1 = MR_2 = MC_{total}$$

Working for the second last line of the equation above we can see why. Given that:

$$\pi = p_1 \cdot q_1 + p_2 \cdot q_2 - (C(q_1 + q_2))$$

We can maximize profit with respect to two variables, q_1 and q_2.

(Noting that p_1 is a function of q_1, as to get p_1 we will plug q_1 into group 1's demand function, and likewise for group 2, and given a total cost function, we can see that the only unknowns in that equation are q_1 and q_2).

To do this maximization we take the derivitive with respect to each variable seperately and set it equal to zero. Thus we will have two equations in two unknowns.

$$\max_{q_1}(\pi) \implies \frac{d(p_1 \cdot q_1)}{dq_1} - \frac{d(C(q_1 + q_2))}{dq_1} = (MR_1 - MC) = 0$$

and

$$\max_{q_2}(\pi) \implies \frac{d(p_2 \cdot q_2)}{dq_2} - \frac{d(C(q_1 + q_2))}{dq_2} = (MR_2 = MC) = 0$$

This gives us a solution for both q_1 and q_2, and so p_1 and p_2.

Let's see this in practise. Suppose:

$$p_1 = 10 - 2q$$
$$p_2 = 8 - q$$
$$C = 10 + \frac{q^2}{2}$$

Then

$$\pi = p_1 \cdot q_1 + p_2 \cdot q_2 - C = (10 - 2q_1) \cdot q_1 + (8 - q_2) \cdot q_2 - \left(10 + \frac{(q_1 + q_2)^2}{2}\right)$$

And

$$\max_{q_1}(\pi) \implies \underbrace{10 - 4q_1}_{MR_1} - \underbrace{(q_1 + q_2)}_{MC} = 0$$
$$\max_{q_2}(\pi) \implies \underbrace{8 - 2q_2}_{MR_2} - \underbrace{(q_1 + q_2)}_{MC} = 0$$

Rearranging these we two equations respectively for q_1 we get

$$q_1 = 2 - \frac{q_2}{5}$$
$$q_1 = 8 - 3q_2$$

$$\therefore q_2 = \frac{30}{14} \quad \text{and} \quad q_1 = 8 - 3 \cdot \frac{30}{14} = \frac{22}{14}$$

To recap, if marginal cost is constant you can set $MR = MC$ for each group seperately. However, is marginal cost is not constant, that is if marginal cost is a function of quantity and so is increasing or decreasing in quantity sold, then this short-cut approach will give you the wrong answer. Instead you must maximize the profit function with respect each group's quantity seperately and then solve the simultaneous equations. This is exactly equivalent to setting the marginal revenue for each group equal to marginal cost, and so to each other.

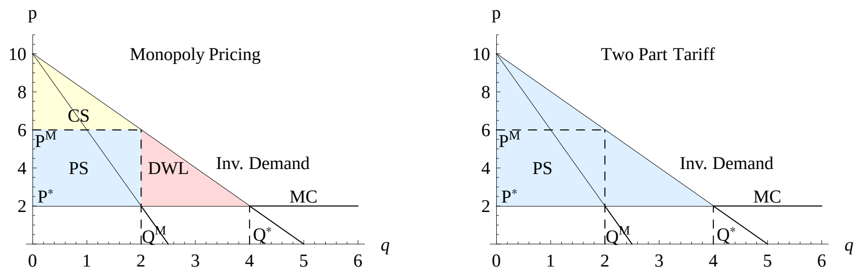
Two Part Tariffs

Two part tariffs use two prices: one price for every unit purchased (i.e. a regular price), and one price to "enter the store" or to have the right to buy any units (i.e. an entry fee). We often denote the entry fee with the Greek letter phi (ϕ), and we use the letter p for the regular price as per usual.

Two part tariffs are very simple, and you already have all of the tools that you need to solve them. There are two steps:

1. Price using competitive pricing ($p = mc$) to get the efficient quantity sold to the consumer
2. Extract all of the consumer's surplus using first degree price discrimination ($\phi = CS$)

Using this scheme you do not make any profits from the individual units sold if marginal cost is constant, however, you get $\Pi = N \cdot \phi$, where N is the number of consumers you serve.



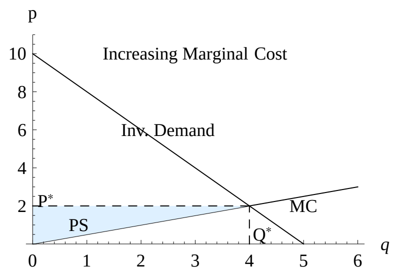
Comparing this to monopoly pricing we have the following results:

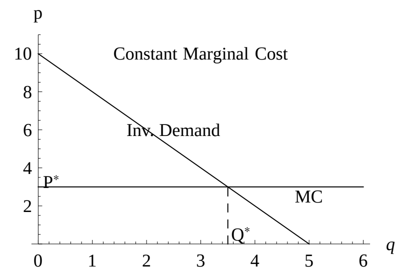
- Two-part tariffs are efficient - they have no deadweight loss and every customer purchases until their willingness to pay for an addition unit is equal to the marginal cost
- Two-part tariffs yield higher profits for the monopolist than monopoly pricing. In the graph, the profit contribution under a two part tariff is equal to the sum of the old consumer surplus + the deadweight loss (DWL) + the old producer surplus ($\phi = CS_M + DWL_M + PS_M$). However, they can only be applied to goods that can not be arbitraged (i.e. goods for immediate consumption).

The canonical example of a two-part tariff is a (private) gold club. The membership fee is the entry fee, and the price per round is the unit price.

Competitive Markets

Regardless of whether we are considering the short-run or the long run, in competitive markets it is always true that **price equals marginal cost** ($p = mc$). If marginal cost slopes up then in the short run firms can make a profit in a competitive environment. However, If marginal cost is constant the issue of short run versus long run is moot, firms always make no profits.





To determine the quantity that each firm produces in the short run when marginal cost slopes up we need simply set $p = mc$, as price is given and mc is in terms of quantity.

For example, if $mc = 2q$ and $p = 10$ then:

$$p = mc \quad \implies 10 = 2q \quad \therefore q = 5$$

To determine the total quantity produced in the market we do something different - we know that all price quantity pairs must lie on the demand line, so we plug the price into aggregate demand.

For example if $Q = 10,000 - 500P$ and $P = 10$ as before, then $Q = 10,000 - 500 \cdot 10 = 5,000$.

To determine the number of firms in the market, simply determine how many firms each producing q would be needed to produce an aggregate of Q:

$$N \cdot q = Q \quad \therefore N = \frac{Q}{q}$$

You may well know the most famous relationship in economics already:

Supply = Demand

Put differently this says that where the supply function and the (inverse) demand function intercept, there is a single[2] (q,p) pair that is the competitive market equilibrium. We found this point by equating a firm's supply curve (which is its marginal cost curve) and the demand curve (i.e. the price). We could also have found it by equating aggregate supply and aggregate demand.

To aggregate a supply curve you must put it in the form $q = \ldots$. As with the aggregation of demand, we **always aggregate quantities** and never aggregate prices. Each firm's supply curve is given by its marginal cost curve ($p = mc$).

[2] In MBA economics you don't need to worry about there being more than one, but generally some very benign assumptions about the shape of demand and supply curves are enough to gaurantee this

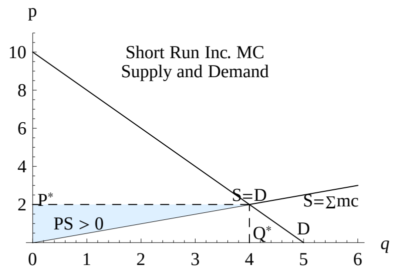
We sum supply curves over firms, so that if firms have identical supply curves then:

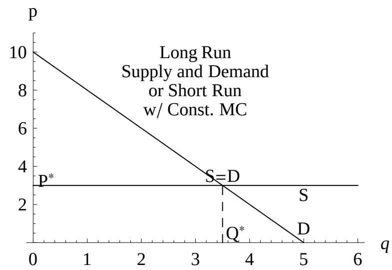
$$Q = N \cdot q$$

If, for example, $mc = 2q$ then:

$$p = 2q \implies q = \frac{p}{2} \quad \therefore Q = N \cdot \left(\frac{p}{2}\right)$$

If marginal cost is flat, or in the long run (where supply curves are flat), then this aggregation is meaningless. For example if $mc = 2$ then mc is 2 regardless of q and regardless of the number of firms in the market. Furthermore as $p = mc$, p is also 2 regardless of q and regardless of the number of firms in the market. For this reason, long run supply curves are always simply the price - a constant flat line. All firms are price takers making zero profit - if a firm prices above marginal cost is will get no customers, and it can not price below marginal cost as it will be making a loss on every unit sold. The only possibility is to price at marginal cost, and this price is irrespective of the quantity produced or sold.





Even if marginal cost slopes upwards, in the long run there will be no profits as long as there is free entry. The logic is that if firms were making profits then entrepreneurs would recognize this and enter the market until the market no longer made profits. Our defining condition for a long run equilibrium is that profits are zero. For convenience we will often state this as price equals average cost (ac). These two statements are equivalent:

$$\pi = 0 \implies R - C = 0 \implies R = C \implies \frac{R}{q} = \frac{C}{q}$$

$$\therefore \frac{p \cdot q}{q} = \frac{C}{q} \implies p = ac$$

This second condition, when combined with our "always true in a competitive environment" condition of $p = mc$, gives us two equations in two unknowns (q, p). That is, given these two equations, and nothing else, we can solve for a firm's equilibrium price-quantity pair.

To solve for the equilibrium quantity equate $mc = ac$, and to solve for the equilibrium price plug the solution back in to either equation. To determine the long run number of firms, again plug the price into the aggregate demand function to get the aggregate quantity, and again solve $N = \frac{Q}{q}$.

Let's practice this with a simple example. Suppose:

$$
\begin{aligned}
C =& 100 + 10q^2 \\
Q =& 5000 - 10p
\end{aligned}
$$

Then:

$$mc = 20q \text{ and } ac = \frac{100}{q} + 10q$$

$$\text{Setting } mc = ac \quad 20q = \frac{100}{q} + 10q \quad \therefore q^* = 10$$

$$\text{Plugging in to } p = mc \quad p^* = 20q^* = 20 \cdot 10 = 200$$

$$\text{Plugging in to } Q^* = 5000 - 10 \cdot 200 \quad Q = 3000$$

$$\text{Plugging in to } N^* = \frac{Q}{q} \quad N = \frac{3000}{10} = 300$$

Note that we denote competitive equilibrium results (prices, quantities, etc) with an asterisk (*).

If we were told that the short run price was $p_{sr}300$, then in the short run the quantity, q_{sr} produced was:

$$p_{sr} = mc \implies 300 = 20q_{sr} \implies q_{sr} = 15$$

$$\pi = p \cdot q - C = 300 \cdot 15 - (100 + 10 \cdot 15^2) = 1950 > 0$$

$$N_{sr} = \frac{Q}{q} = \frac{5000 - 10 \cdot 300}{15} = \frac{2000}{15} = 13\frac{1}{3}$$