Baron & Ferejohn (APSR 1989)

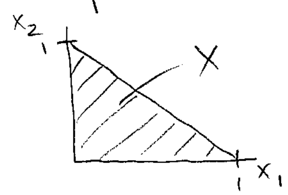
- Setup

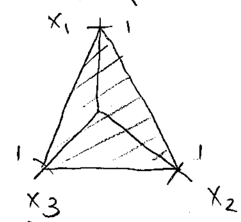
Players: n members $i=1,\dots,n$ $\quad n$ odd

Preferences: $u_i(x) = x_i$

$u^j(x^k, t) = \delta^t x_j^k$

Proposals: Simplex X n-dim s.t. $\sum_i x_i \leq 1$





Strategies: $S_t^i : H_t \rightarrow \{x, v(x)\}$

Information: complete

Equilibrium concept: ① SPNE ≡ equilibrium strategies are equilibria in every proper subgame

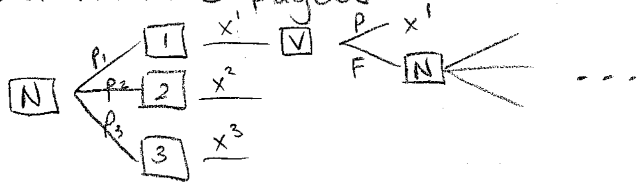
② majority rule

③ elimination of weakly dominated strategies

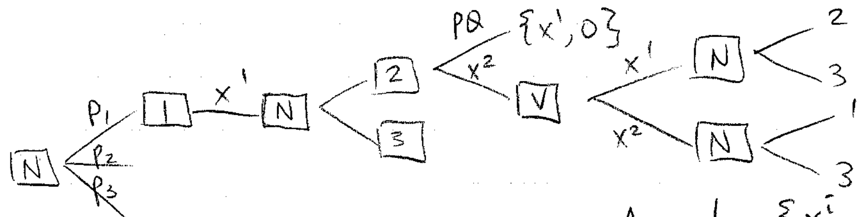
- Notation

$i=1,\dots,n$	legislator
$x=(x_1, x_2, \dots, x_n)$	proposals
$x^i=(x_1^i, x_2^i, \dots, x_n^i)$	i's proposal
$\delta \in (0,1)$	discount factor
$u^j(x^k, t) = \delta^t x_j^k$	utility of player j if legislature adopts proposal x^k in session t
$p^i \in (0,1)$ $\sum_i p^i = 1$	probability i is recognized
$q = \vec{0} = (0,\dots,0)$	status quo
h_t	history of who moved at each period
H_τ	set of all possible histories
$v_i(t,g)$	value to i of subgame g after t session.
$\delta v_i(t,g)$	continuation value to i of subgame g after t sessions
v_i	value of the game at the beginning to player i

- Game Form
  - Closed Rule: 3 players (no amendments)



  - Open Rule: 3 players (amendments)



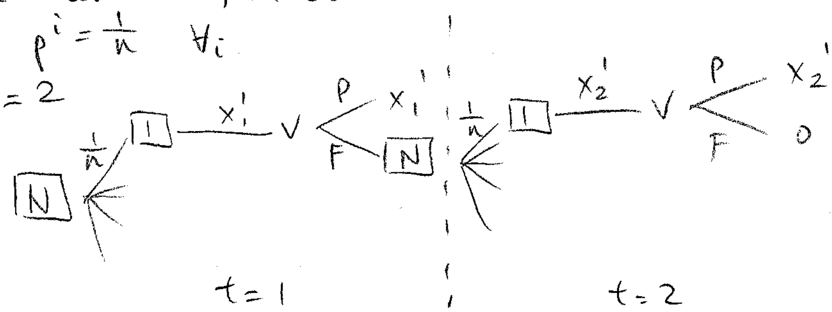
Agenda: $\{x^i, 0\}$

- $\boxed{\text{Example 1}}$: Finite legislature, Closed Rule

  n-members, n odd

  $p^i = \frac{1}{n} \quad \forall i$

  $T = 2$



  - Proposition: The following is "unique" equilibrium:

    ⓐ (i) if recognized in 1st session, propose:

    $$\begin{cases} \delta/n \text{ to some } \frac{n-1}{2} \text{ others} \\ 0 \text{ to some } \frac{n-1}{2} \text{ others} \\ \text{keep rest for self } \left(1 - \frac{\delta(n-1)}{2n}\right) \end{cases}$$

    (ii) if recognized in 2nd session, propose:

    $$x^i_{i2} = 1, \; x^i_{j2} = 0 \quad \forall i \neq j$$

    ⓑ vote for proposals in 1st session if $\delta/n \leq x^i_{1j}$

    vote for any proposal in 2nd session

  Proof: Backward Induction

  $V_i(2, g) = 0$

  Since $x^i_j \geq 0$, then $\forall i$ vote for any proposal

  Given above, what will a recognized member propose?

  $$\max_{x_i} u_i(x_i \mid x_i \geq 0 \; \forall i) \Rightarrow x^i_{i2} = 1, \; x^i_{j2} = 0 \quad \forall i \neq j$$

Then $V_i(1, g) = 1/n \ \forall g$ and $\delta V_i(1, g) = \delta/n$

Recall member j votes for a proposal in period t from i iff $x^i_{jt} \geq \delta V_j(t, g)$

(i.e. proposal is better than the continuation value)

Therefore, propose δ/n to $\frac{n-1}{2}$ others

$1 - \frac{\delta(n-1)}{2n}$ to self

Is to self better than to others?

$$1 - \frac{\delta(n-1)}{2n} \geq \frac{\delta}{n}$$

$$2n - \delta(n-1) \geq 2\delta$$

$$2n - \delta n \geq \delta$$

$$2n \geq \delta(n+1) \quad \text{always holds}$$

Thus, always better off to propose and this is an equilibrium

- Result: ① minimum winning coalitions (mwc)

  ② member recognized in period 1 receives the largest payoff

  equilibrium outcome: propose, accepted, end

  "proposal power" or "agenda power"

  ③ $V_i = \frac{1}{n}\left(1 - \frac{\delta(n-1)}{2n}\right) + \frac{n-1}{2n}(\delta/n) + \frac{n-1}{2n}(0) = 1/n$

  ex ante value of the game

  game is "fair"

- Claim 1: $T = 3$, $p^i = \frac{1}{n} \ \forall i$

  Last session: $V_i(3, g) = 1/n$

  Same game as $T=2$

  For all finite games, same result

- Claim 2: $T = 2$, $p^i \neq p^j \ i \neq j$

  ① $n = 3$, $p^1 = 1/3 + \varepsilon$ $p^2 = 1/3$ $p^3 = 1/3 - \varepsilon$

$$\left.\begin{aligned} V_1 &= \tfrac{1}{3}[1 - \delta/3] \\ V_2 &= \tfrac{1}{3}[1 - \delta/3] + \tfrac{1}{3}[\delta/3] \\ V_3 &= \tfrac{1}{3}[1 - \delta/3] + \tfrac{2}{3}[\delta/3] \end{aligned}\right\} \text{if } \delta = 1 \quad \begin{aligned} V_1 &= \tfrac{2}{9} \\ V_2 &= \tfrac{1}{3} \\ V_3 &= \tfrac{4}{9} \end{aligned}$$

  ② $n = 3$, $p^1 = 3/4$ $p^2 = 1/4$ $p^3 = 0$

  $V_1 = 3/4$ $V_2 = 1/4$ $V_3 = 0$

with asymmetric probabilities, trade off b/t value of being cheapest vs. value of being recognized

- $\boxed{\text{Example 2}}$ = Infinite legislature, Closed Rule
  - Proposition 2:
    "Folk Theorem": If discounting is not too severe, with punishments that are credible, any $x \in X$ is supportable as an equilibrium
  - Definition: structurally equivalent
    Two subgames are <u>structurally equivalent</u> if
    (i) the agenda at the initial node of the subgames are identical
    (ii) sets of members who may be recognized at the next recognition stage are identical
    (iii) strategy sets are the same
  - Definition: stationary
    An equilibrium is <u>stationary</u> if the actions all players take for each structurally equivalent subgame are the same
    i.e. "history-independent"
    continuation values are constant
  - <u>Proposition 3</u>:
    For all δ in an infinite session, majority rule, n (odd) members, legislature with a closed rule, the following is a <u>unique</u> stationary SPNE:
    (i) recognized member proposes:

$$\begin{cases} \delta/n & \text{to some } \frac{n-1}{2} \text{ others} \\ 0 & \text{to some } \frac{n-1}{2} \text{ others} \\ 1-\frac{\delta(n-1)}{2n} & \text{to self} \end{cases}$$

    (ii) member j votes for proposals in which $x_j \geq \delta/n$

Assume $p^i = 1/n$

<u>Proof</u>: $\forall g$ starting at N (identical)

Let V_i = value of a subgame for i

$V_i = V_i(t, g) \ \forall t$ by stationary assumption

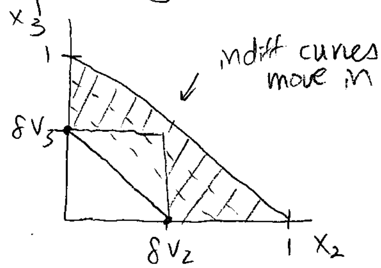
To get a winning proposal, a recognized member must offer δV_j to j for j to accept

For an unrecognized member,

$$V_j = \underset{\substack{\uparrow \\ \text{recognized}}}{\frac{1}{n}} [\underset{\substack{\uparrow \\ \text{recognition value}}}{1-\frac{n-1}{2}\delta V_j}] + \underset{\substack{\text{not recognized} \\ \text{in coalition}}}{\frac{n-1}{2n}} \underset{\substack{\uparrow \\ \text{in coalition value}}}{\delta V_j} \quad , \ j = 1, \ldots, n$$

Thus, $v_j = 1/n$ and $\delta v_j = \delta/n$

- Result: ① mwc (minimum winning coalition)
  ② proposal power $x_i^1 \geq 1/2 \geq \delta/n$
  ③ fairness $v_i = 1/n$
  ④ Graphically



$\max 1 - x_2^1 - x_3^1$ s.t. $x \notin Z$

⑤ Comparative Statics

$$x_i^i = 1 - \frac{\delta(n-1)}{2n}$$

$$\frac{\partial x_i^i}{\partial \delta} = -\frac{n-1}{2n} < 0$$ patience reduces proposal power

$$\frac{\partial x_i^i}{\partial n} = -\frac{\delta}{2} + \frac{\delta(n-1)}{2n^2} = \frac{-\delta(n^2-n-1)}{2n^2} < 0$$

larger legislatures reduce proposal power

$$\frac{\partial\left(1 - \frac{\delta(n-1)}{2n} - \frac{\delta}{n}\right)}{\partial n} = \frac{\delta}{n^2} > 0$$

worse off the larger the size of the party (coalition)

- $\boxed{\text{Example 3}}$: Open Rule

Choose universalism vs. mwc

universalism: pay everybody but just 1/n for self (definitely pass)

mwc: pay minimum for everybody else, lots for self (low chance of passing)

result: choose size of coalition

System of 4 equations

in/out coalition × recognized / unrecognized