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Relaxing Price Competition Through Product Differentiation

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The notion of a Perfect Equilibrium in a multi-stage game is used to characterize industry equilibrium under Monopolistic Competition, where products are differentiated by quality.

Central to the problem of providing adequate foundations for the analysis of monopolistic competition, is the problem of describing market equilibria in which firms choose both the specification of their respective products, and their prices. The present paper is concerned with a—very particular—model of such a market equilibrium. In this equilibrium, exactly two potential entrants will choose to enter the industry; they will choose to produce differentiated products; and both will make positive profits.

1. THE EQUILIBRIUM CONCEPT

Our present analysis is based on a three stage non-cooperative game. In the first stage, firms choose whether or not to enter the industry. At the end of the first stage, each firm observes which firms have entered, and which have not. In the second stage each firm chooses the quality of its product. Then, having observed its rivals' qualities, in the final stage of the game, each firm chooses its price. This three stage process is intended to capture the notion that the price can in practice be varied at will, but a change in the specification of a product involves modification of the appropriate production facilities; while entry to the industry requires construction of a plant.

The strategies of firms specify actions to be taken in each of the three stages.

Thus a (pure) strategy takes one of two forms, "don't enter", or else "enter; choose a level of quality, dependent on the number of firms who have entered; and set price, dependent both on the number of entrants and on the quality of their respective products".

The payoffs will be defined in terms of a model of consumer choice between the alternative products, in Section 2 below. They will be identified with the profit earned by the firm, less a "cost of entry" of $\varepsilon > 0$, for those who enter; and zero for non-entrants.

We may now define the solution concept. As in any non-cooperative game, we might investigate the set of Nash Equilibria. Here, as is often the case, that set may be very large. We therefore introduce the now familiar concept of a Perfect Equilibrium (Selten (1975)).

An n-tuple of strategies is a Perfect Equilibrium in this three stage game, if, after any stage, that part of the firms' strategies pertaining to the game consisting of those stages which remain, form a Nash Equilibrium in that game.

It follows immediately from this that, after any stage, that part of the firms' strategies pertaining to the game consisting of those stages which remain, in fact form a Perfect Equilibrium in that game.

Thus, for example, when firms have decided whether to enter, and have chosen their qualities, we require that their price strategies are a Nash Equilibrium, i.e. a non-cooperative price equilibrium, in the single remaining stage of the game.

To study such a Perfect Equilibrium, we begin, therefore, by analysing the final stage of the game—being the choice of price, given the number of entrants and the qualities of their respective products (Section 2). We will then proceed, in Section 3, to examine the choice of quality by firms, and in Section 4 we consider the entry decision. Section 5 contains a summary of the argument, and develops some conclusions.

2. PRICE COMPETITION

Consider a number of firms producing distinct, substitute goods.[1] We label their respective products by an index $k=1, \ldots, n$ where firm k sells product k at price p_k.

Assume a continuum of consumers identical in tastes but differing in income; incomes are uniformly distributed, viz. the density equals unity on some support $0<a \leqq t \leqq b$.

Consumers make indivisible and mutually exclusive purchases from among these n goods, in the sense that a consumer either makes no purchase, or else buys exactly one unit from one of the n firms.[2] We denote by $U(t, k)$ the utility achieved by consuming one unit of product k and t units of "income" (the latter may be thought of as a Hicksian "composite commodity", measured as a continuous variable); and by $U(t, 0)$ the utility derived from consuming t units of income only.

Assume that the utility function takes the form

$$U(t, k)=u_k \cdot t \tag{1}$$

with $u_0<u_1<\cdots<u_n$ (i.e. the products are labelled in increasing order of quality).

Let

$$C_k=\frac{u_k}{u_k-u_{k-1}}$$

(whence $C_k>1$). Then we may define the income level t_k such that a consumer with income t_k is indifferent between good k at price p_k and good $k-1$ at price p_{k-1}, viz.

$$U(t_k-p_k, k)=U(t_k-p_{k-1}, k-1)$$

whence

$$t_1=p_1C_1$$

and

$$t_k=p_{k-1}(1-C_k)+p_kC_k. \tag{2}$$

This is easily checked by reference to (1).

Now it follows immediately on inspection of (1) that consumers with income $t>t_k$ strictly prefer good k at price p_k to good $k-1$ at p_{k-1}, and conversely; whence consumers are partitioned into segments corresponding to the successive market shares of rival firms.

Assuming zero costs the profit (revenue) of the k-th firm is:

$$\begin{aligned}
R_1&=\begin{cases} p_1(t_2-a) & t_1\leqq a \\ p_1(t_2-t_1) & t_1\geqq a \end{cases}\\
R_k&=p_k(t_{k+1}-t_k), \qquad 1<k<n \\
R_n&=p_n(b-t_n).
\end{aligned} \tag{3}$$

Now, at equilibrium (if it exists), it follows trivially that the top quality product will enjoy a positive market share; moreover if any product has zero market share, so also do all lower quality products.

Now where n products co-exist at equilibrium (i.e. each of these n goods has a positive market share) the first order necessary conditions for profit (revenue) maximization take the form,

for $k = 1$,

$$t_2 - a - p_1(C_2 - 1) = 0 \qquad t_1 \leqq a$$

$$t_2 - t_1 - p_1[(C_2 - 1) + C_1] = 0 \qquad t_1 \geqq a$$

for $k = 2, \ldots, n-1$,

$$t_{k+1} - t_k - p_k[(C_{k+1} - 1) + C_k] = 0 \tag{4}$$

for $k = n$,

$$b - t_n - p_n C_n = 0.$$

We may now proceed to establish:

Lemma 1. *Let $b < 4a$. Then for any Nash Equilibrium involving the distinct goods $n, n-1, \ldots, 1$ at most two products (the top two) have a positive market share at equilibrium.*

Proof. Assume that there exists a Nash Equilibrium in which three or more products have a positive market share at equilibrium. From inspection of the necessary conditions for profit maximization (4), and remembering $C_k > 1$, it follows that, for $k > 1$, and $k = n$, respectively, by rewriting the first order conditions and using the definition of t_k,

$$t_{k+1} - 2t_k - p_k(C_{k+1} - 1) - p_{k-1}(C_k - 1) = 0;$$

$$b - 2t_n - p_{n-1}(C_n - 1) = 0$$

whence

$$b > 2t_n, \qquad t_{k+1} > 2t_k$$

whence

$$4t_{n-1} < b.$$

Now by assumption $b < 4a$, so that $t_{n-1} < a$ i.e. the top two firms cover the market. Thus equilibrium involves at most two products. ||

The idea here is that price competition between "high quality" products drives their prices down to a level at which not even the poorest consumer would prefer to buy certain lower quality products even at price zero. Clearly, the number of products which can survive at equilibrium depends on the distribution of income. Lemma 1 provides a restriction, that $b < 4a$, which is sufficient to limit this number to *at most* two; we shall in fact be concerned with this case in what follows.

It will be convenient at this point, then, to cite the special form of the revenue functions and the first order conditions for the case where $n = 2$, i.e. where exactly two firms enjoy a positive market share.

We define

$$V = \frac{u_2 - u_0}{u_2 - u_1} = \frac{C_2 - 1}{C_1} + 1 \tag{5}$$

being a measure of the relative qualities of goods 1 and 2, and the residual good 0.

Applying equation (2) we have here that

$$p_1 = \frac{t_1}{C_1} \quad \text{and} \quad p_2 = \frac{t_2 + t_1(V-1)}{C_2}. \tag{6}$$

Using equations (5), (6) we may re-write the first order conditions for profit maximization in terms of t_1, t_2, V, viz.

firm 1:

$$\begin{aligned} t_2 &= a + t_1(V-1) \qquad t_1 \leqq a \\ t_2 &= t_1(V+1) \qquad t_1 \geqq a \end{aligned} \tag{7}$$

firm 2:

$$b - 2t_2 = t_1(V-1). \tag{8}$$

We identify three regions as illustrated in Figure 1. For a certain range of p_2 chosen by firm 2, the optimal reply of firm 1 leads to an outcome (t_1, t_2) in region II, i.e. $t_1 = a$, $aV \leqq t_2 \leqq a(V+1)$. Over this range firm 1 leaves its price constant as p_2 varies; at the price p_1 it chooses, the poorest consumer is just willing to buy good 1. Firm 1 faces a demand schedule which is kinked at this price level (given p_2); and either raising or lowering price reduces revenue. Thus we have a corner solution, and the equalities of (7) are replaced by a pair of inequalities.

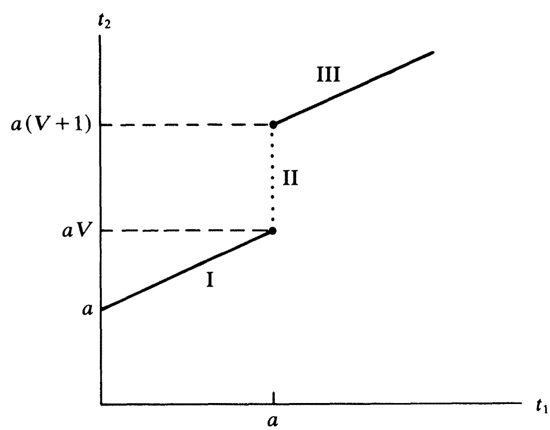

FIGURE 1

The first order conditions for profit maximization by firm 1

Equation (8) describes a decreasing function of t_1,

$$t_2 = \tfrac{1}{2}[b - t_1(V-1)].$$

The intersection of (7), (8) defines the unique equilibrium pair (t_1, t_2), and so the equilibrium pair (p_1, p_2).

Whether the solution lies in region I, II or III depends on where the decreasing function (8) cuts the vertical $t_1 = a$, viz.

$$\text{region I if} \qquad V \geqq \frac{b+a}{3a}$$

$$\text{region II if} \qquad \frac{b+a}{3a} \geqq V \geqq \frac{b-a}{3a}$$

$$\text{region III if} \qquad V \leqq \frac{b-a}{3a}.$$

Thus the solution lies in region I when the qualities are "close", and in region III when the qualities are "far apart".

If the solution lies in region III, then $t_1 > a$ and some consumers purchase neither good. If the solution lies in region II then all consumers purchase one or other good—the market is "covered", and the poorest consumer is indifferent between buying the low quality product 1 or not. In region I the market is again covered, but now the poorest consumer strictly prefers to purchase product 1.

Moreover, we note from equations (7), (8) that,

in region I:

$$t_1 = \frac{b-2a}{3(V-1)}, \qquad t_2 = \frac{b+a}{3} \tag{9}$$

while

in region II:

$$t_1 = a, \qquad t_2 = \tfrac{1}{2}[b - a(V-1)]. \tag{10}$$

We are now in a position to strengthen our earlier result.

Lemma 2. *Let $2a < b < 4a$. Then of any n firms offering distinct products, exactly two will have positive market shares at equilibrium. Moreover, at equilibrium, the market is covered (i.e. the equilibrium is not in region III).*

Proof. From Lemma 1 only two goods (at most) will survive with positive market shares and positive price. Hence we may write the equations for this case as developed above.

We note from Figure 1 that the decreasing function (8) lies above a for $t_1 = 0$ (Note $t_2(0) = b/2 > a$).

Hence the two functions (7), (8) intersect at a point such that $t_1 > 0$, $t_2 > a$, so that the two products coexist with positive market shares.

To verify that this solution is indeed an equilibrium, it must further be shown that the second order conditions are satisfied, i.e. the revenue function of firm 1, given p_2, is concave, over all p_1; and conversely. This may be verified in a straightforward manner. Moreover, since $b < 4a$, $(b-a)/3a < 1$ and the condition for the solution to be in region III cannot be met (as $V > 1$) so the market is covered. ||

From this point forward we shall assume that $2a < b < 4a$.

The preceding discussion establishes the existence of a unique price pair which forms a Nash Equilibrium in prices, for any two distinct levels of product quality. Moreover, both firms enjoy strictly positive revenue. If on the other hand the firms choose the same level of quality, our use of a non-cooperative price equilibrium ensures that both prices become zero (the Bertrand duopoly case); so that both firms have revenue zero at equilibrium. In either case, the equilibrium vector of payoffs (revenues) is uniquely determined via our preceding discussion.

We now turn to the case where more than two firms enter the industry. Still assuming, as always, that $2a < b < 4a$, we distinguish two cases. If one firm has a quality lower than either of its rivals, it has a zero market share, and so revenue zero, as shown in Lemma 1. If two (or three) firms have an equal lowest quality, then the price of this lowest quality product is zero at equilibrium (again from the usual Bertrand argument). In either case, any firm setting the lowest, or equal lowest, quality, has revenue zero at equilibrium. Thus, the equilibrium vector of payoffs (revenues) of the firms present in the industry is uniquely determined via our preceding characterization.

3. COMPETITION IN QUALITY

We now turn to the preceding stage of the process, in which firms choose quality. Let k denote the number of firms who have entered. We introduce the notation G^k to denote the 2-stage game in which quality is first chosen, and then price.

Finding a perfect equilibrium in G^k is equivalent to finding a Nash equilibrium in qualities, the payoffs arising from any vector of qualities being defined by the (unique) equilibrium vector of revenues in the "choice of price" game of the preceding section.

We suppose for the moment that the number of firms is exactly two, deferring the question of further "potential entrants" until later.

Each firm chooses a level of quality, being a value u_i, $u_0 < u_i < \bar{u}$, where $\bar{u}$ is an exogenously given upper bound on quality.

We introduce the notation $R(u; v)$ to denote the revenue of a firm whose product is of quality u, its rival's product being of quality v, at a Nash equilibrium in prices.

We will establish the existence of an equilibrium involving differentiated products, as a consequence of two properties of the revenue function $R(u; v)$. The first property, stated in Lemma 3, is that, at equilibrium, the revenue of the firm offering the higher quality product is greater. The second property (Lemma 4) states that the revenue of both firms increases as the quality of the better product improves. The latter property reflects the effect of the lessening of price competition as qualities diverge, and is the key to the existence of an equilibrium with differentiated products in the present analysis. (This runs counter to the classic Hotelling "Principle of Minimal Differentiation", of course (Hotelling (1929), d'Aspremont, Jaskold Gabszewicz and Thisse (1979)).

Lemma 3. *For any two qualities $u > v$, the top quality firm enjoys greater revenue than its rival, i.e.*

$$R(u; v) > R(v; u).$$

Proof. Let the pair of prices p, q for u and v respectively be a Nash Equilibrium in prices. Trivially, $p > q$. But one strategy open to the top firm is to set its price equal to q, (whereupon the low quality firm has sales zero) and its sales clearly exceed those of its rival in the initial equilibrium. Hence our result follows immediately. ‖

Lemma 4. *The revenues of both firms increase as the quality of the better product improves, i.e.*

$$R(v; u) \text{ and } R(u; v) \text{ are increasing in } u \text{ for } u \geqq v.$$

Proof. We establish the result separately for the two cases where the outcome is in region I, and in region II, respectively.

We begin by writing down the revenue of both firms in region I. We have from (9) that the revenue of firm 1 is

$$\begin{aligned} R(u_1; u_2) &= p_1(t_2 - a) = \frac{t_1}{C_1}\left(\frac{b-2a}{3}\right) \\ &= \left(\frac{b-2a}{3}\right)^2 \frac{1}{(V-1)C_1} = \left(\frac{b-2a}{3}\right)^2 \frac{u_2 - u_1}{u_1} \end{aligned} \tag{11}$$

while the revenue of firm 2 is

$$R(u_2; u_1) = p_2(b - t_2) = \left(\frac{2b-a}{3}\right)^2 \frac{1}{C_2} = \left(\frac{2b-a}{3}\right)^2 \left(\frac{u_2 - u_1}{u_2}\right). \tag{12}$$

Both these expressions increase with u_2, for $u_2 \geqq u_1$, whence our result follows.

In region II we have from (10) that

$$R(u_1;\, u_2) = p_1(t_2 - a) = \frac{a[b - a(V+1)]}{2C_1}$$

$$R(u_2;\, u_1) = p_2(b - t_2) = \frac{[b + a(V-1)]^2}{4C_2}.$$

That $R(u_1; u_2)$ increases in u_2 follows on noting that V falls as u_2 increases (note C_1 is independent of u_2).

For $R(u_2; u_1)$, we note that, by definition of C_1, C_2, V we have

$$C_2 = C_1(V-1) + 1$$

whence the logarithmic derivative of $R(u_2; u_1)$ w.r.t. V is

$$\frac{2a}{b + a(V-1)} - \frac{C_1}{C_1(V-1)+1} = \frac{aC_1(V-1) + 2a - C_1 b}{[b + a(V-1)][C_1(V-1)+1]}$$

where, for region II,

$$\frac{b-a}{3a} \leqq V \leqq \frac{b+a}{3a}.$$

The denominator is positive since $V > 1$ so the sign coincides with that of the numerator. We wish to establish therefore, that the numerator is negative; but since it is a linear increasing function of V it suffices to show that it is negative when V takes its maximum value in region II i.e. $V = (b+a)/3a$.

But here the numerator is

$$C_1\left(\frac{b-2a}{3}\right) + 2a - C_1 b = 2a - \tfrac{2}{3}C_1(a+b) < 2a(1 - C_1) < 0$$

where we have used the fact that $b > 2a$. Thus $R(u_2; u_1)$ decreases with increasing V, i.e. is increasing in u_2. ||

We now define the "optimal reply from below" as follows. Let one firm set quality u. Then, of all qualities on the restricted range $[u_0, u]$ we choose that level v which maximizes the revenue $R(v, u)$. Since $R(v; u)$ is continuous in v it follows that for any u, $R(v; u)$ takes a maximum over v in the closed set $[u_0, u]$. Moreover, for $v = u$, $R(v; u) = 0$, while for $u_0 < v < u$, $R(v; u) > 0$; so that the maximum is attained at a quality strictly less than u.

We define the set[3] of optimal replies

$$\rho(u) = \{v \mid R(v; u) = \max R(s; u);\ u_0 \leqq s \leqq u\}$$

and our preceding remarks imply that $\rho(u) \neq \emptyset$ and $u \notin \rho(u)$ for $u_0 < u$.

We may now establish[4]

Proposition 1. *The game G^2 has a perfect equilibrium in pure strategies; the outcome involves distinct qualities, and both firms earn positive revenue (profits) at equilibrium.*

Proof. We demonstrate the existence of such an equilibrium as follows. Choose a $v \in \rho(\bar{u})$. Then we will show that the pair $(\bar{u}, v)$ is a Nash Equilibrium in the "choice of quality" game, with the payoffs defined as the revenue obtained in the "choice of price" game of the preceding sections, and so is a Perfect Equilibrium in G^2.

Let the firm setting $\bar{u}$ be labelled 2, and its rival 1. To show that $(\bar{u}, v)$ is a Nash Equilibrium, we note that, given a choice of $\bar{u}$ by firm 2, then the choice of v by firm 1 is optimal, by definition of $\rho(u)$.

To complete our proof we show that, given a choice of v by firm 1, $\bar{u}$ is an optimal choice for firm 2.

We divide the argument into two parts. First note that $\bar{u}$ is preferred to any $u \geqq v$ by virtue of Lemma 4. Secondly, consider the payoff to firm 2 if it chooses any quality u_2 where $u_0 \leqq u_2 < v$.

Then we have

$$R(u_2; v) \leqq R(u_2; \bar{u}) \qquad \text{by Lemma 4.}$$

But

$$R(u_2; \bar{u}) \leqq R(v; \bar{u}) \qquad \text{as } v \in \rho(\bar{u}).$$

While

$$R(v; \bar{u}) \leqq R(\bar{u}; v) \qquad \text{by Lemma 3.}$$

Hence

$$R(u_2; v) \leqq R(\bar{u}; v)$$

and the choice of $\bar{u}$ is indeed optimal for firm 2 as required. ||

We have thus established that with 2 firms present, a Nash Equilibrium in qualities exists, which is a Perfect Equilibrium in the two stage game ("choice of quality, choice of price").

We now consider the outcome if $k > 2$ firms are present. We aim to show here, (i) that the choice of $\bar{u}$ by all firms is a Nash Equilibrium,[5] and (ii) that for *any* Nash Equilibrium, all firms have revenue zero. (Up to this point we have confined our attention to equilibria in pure strategies. In fact the proof of (ii) extends trivially to mixed strategies, and we will establish the result in this more general setting below.)

Proposition 2. (i) *The game G^k, $k > 2$ has a Nash Equilibrium*

$$u_i = \bar{u}, \; 1 \leqq i \leqq k.$$

(ii) *For every Nash Equilibrium of G^k the payoff for each firm is zero.*

Proof. (i) Suppose all firms but one choose $\bar{u}$. Then at least 2 firms sell an identical product of quality $\bar{u}$; following the familiar Bertrand argument for a non-cooperative price equilibrium between two firms selling an identical product, we have immediately that each of these firms sets price zero. Hence our remaining firm earns payoff zero for any choice $u \leqq \bar{u}$; for either its price is zero (at $u = \bar{u}$) or its sales are zero (at $u < \bar{u}$). Hence G^k has a Nash Equilibrium, $u_i = \bar{u}$, $1 < i < k$.

(ii) In order to establish this, we show that in every Perfect Equilibrium at least two firms adopt the pure strategy $\bar{u}$; whence the result follows immediately.

Let μ^i be a probability measure on $[u_0, \bar{u}]$ and let $\{\mu^i\}$ be a Nash Equilibrium for G^k. Let V_i be the lim inf of the support of μ^i. Assume $V_1 \leqq V_2 \leqq \cdots \leqq V_k$, and furthermore assume that if any of the μ^i has an atom at V_1 then we label the firms so as to denote it (or one such firm) as 1.

First we show that the payoff of 1 is zero. If V_1 is an atom of μ^1 then the pure strategy V_1 yields payoff zero to firm 1 (given $\mu^2, \ldots, \mu^k$); for here the probability is zero that firm 1 offers the (sole) highest quality; or the (sole) second highest quality, product, whence from the analysis of the non-cooperative price equilibrium it earns payoff zero.

If, on the other hand, μ^1 does not have an atom at V_1 then there is a descending sequence of points in the support of μ^1 with limit V_1. The payoff of all these points as pure strategies is the same, but it tends to zero in the limit where quality approaches V_1: for the probability of the limit point V_1 being the (sole) highest quality, or the (sole) second highest quality, is zero (none of the μ^i has an atom at V_1). Thus the payoff to firm 1 is zero.

We may now deduce that at least two firms adopt the pure strategy $\bar{u}$. Suppose firstly that none of the strategies $\mu^1, \ldots, \mu^k$ is the pure strategy $\bar{u}$. Then there is a neighbourhood of $\bar{u}$, and an $\varepsilon > 0$, such that with probability $\varepsilon > 0$ none of the firms $2, \ldots, k$ choose a quality in that neighbourhood. Now we have just shown that the payoff to firm 1 is zero; we now note that μ^1 can not be an optimal strategy, for by choosing the pure strategy $\bar{u}$ firm 1 can now achieve a strictly positive payoff.

Thus at least one of the strategies $\mu^1, \ldots, \mu^k$ is the pure strategy $\bar{u}$. Denote it μ^k. Assume that no other firm adopts this strategy. Then there is a neighbourhood of $\bar{u}$, and an $\varepsilon > 0$, such that with probability $\varepsilon > 0$ none of the firms $2, \ldots, k-1$ choose a quality in this neighbourhood. Firm 1 can thus earn a strictly positive payoff by choosing its quality in this interval.

Hence at least two of the $\mu^1, \ldots, \mu^k$ are the pure strategy $\bar{u}$. Hence all payoffs are zero. ||

4. ENTRY

We have now shown how, in the present model, only two firms can survive with positive prices, and positive market shares, at equilibrium; and how the entry of further firms leads to a configuration in which the top quality product is available at price zero, while all firms earn zero revenue (profits).

We now consider the analysis of entry to the industry. We introduce a "small" cost[6] of entry $\varepsilon > 0$; our results in fact are independent of the size of ε. We define the game G^k_ε as the game G^k introduced above, with ε subtracted from all payoffs. Let there be n potential entrants; they play the three stage game E^n_ε as follows. At the first stage each firm decides whether to enter or not; according as the number who choose to enter is k, these k firms then play the game G^k_ε. Those firms who choose not to enter receive payoff zero.

We establish:

Proposition 3. *For any $\varepsilon > 0$ (sufficiently small), and any number $n > 2$ of potential entrants*

(i) *there exists a Perfect Equilibrium in which two firms enter; and in which they produce distinct products, and have positive revenues (profits).*

(ii) *no Perfect Equilibrium exists in which $k > 2$ firms enter.*

Proof. Corresponding to any pair[7] of firms drawn from n potential entrants, given a decision by these two firms to enter, the payoff to each of the other firms from not entering is zero, while the payoff from entering is $-\varepsilon$ by virtue of Proposition 2. This establishes (ii). Where exactly two firms enter however, each earns a positive payoff (since ε is "small"); and then (i) follows immediately from Proposition 1. ||

5. SUMMARY AND CONCLUSIONS

We have here described a perfect equilibrium of a three stage game in which a number of firms choose firstly, whether to entry an industry; secondly, the quality of their respective products, and thirdly, their prices.

At the final stage of the game, in a non-cooperative price equilibrium, there is an upper bound to the number of firms which enjoy positive market shares, at positive prices (production costs being assumed zero). This reflects the fact that competition between the surviving "high quality" products drives their prices down to a point at which not even the poorest consumer prefers the (excluded) low quality products even at price zero. This number reflects inter alia the utility functions of consumers and the shape of the income distribution. We have here taken a particular form of utility function and assumed a uniform distribution of incomes on $[a, b]$ where $2a < b < 4a$; whence our upper bound is 2. It can be shown by extending our discussion in a natural way, that this upper bound rises as the range of incomes increases.

We establish two results which form the core of the analysis.

(a) We show that where the number of firms equals 2, these two firms will choose distinct qualities, and both will enjoy positive profit at equilibrium. The intuitive idea behind this result is that, as their qualities become close, price competition between the increasingly similar products reduces the profit of both firms.

(b) We show that if three or more firms are present, competition in choice of quality drives all firms to set the same "top" level of quality permitted while prices, and so profits, become zero. This reflects the fact that no one of the three firms will now prefer to set its quality lower than that of its two rivals, as it would thereby certainly earn revenue zero at equilibrium.

Combining (a) and (b) and introducing a small cost of entry ε, we deduce that *the only Perfect Equilibrium in the three stage game is one in which exactly two firms enter; in which they produce distinct products, and earn positive profits at equilibrium.* Moreover, this equilibrium configuration is independent of ε.

A natural question concerns the extension of this model to cases where the upper bound on the number of products which can survive exceeds two. This remains an open question; while property (b) generalizes readily, we have not succeeded in generalizing property (a). Our present argument does not generalize in an obvious manner here.

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NOTES

1. The model of consumer choice over alternative products described here follows Jaskold Gabszewicz and Thisse (1979, 1980). These authors analyse a non-cooperative price equilibrium between firms, the quality of whose products is fixed exogenously. This corresponds to the last stage of our present 3-stage process.
2. Thus our consumer buys *either* this product, or that. Contrast Dixit and Stiglitz (1977).
3. In fact a lengthy development shows that the optimal reply from below, $\rho(u)$, is unique, but this is not required for our present purposes.
4. It may be shown indirectly using the Lemma of Roberts and Sonnenschein (1976), that an equilibrium exists in the present model, but the present direct proof is much shorter.
5. We repeat that a perfect equilibrium in G^k is equivalent to a Nash Equilibrium in qualities, the payoffs arising from any vector of qualities being defined by the (unique) equilibrium values of revenue in the "choice of price" game.
6. Trivially, if ε is sufficiently large, no firm will enter.
7. Of course *any* pair of firms may enter. Similarly, in the "choice of quality" stage, we have, corresponding to the equilibrium $(\bar{u}, v)$, its mirror image $(v, \bar{u})$. The question of *which* firm enters, or sets the higher quality, is outside the scope of this type of model.

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