

## The RAND Corporation

---

Monopolistic Competition with Outside Goods

Author(s): Steven C. Salop

Source: *The Bell Journal of Economics*, Vol. 10, No. 1 (Spring, 1979), pp. 141-156

Published by: The RAND Corporation

Stable URL: <http://www.jstor.org/stable/3003323>

Accessed: 21/10/2008 15:24

---

Your use of the JSTOR archive indicates your acceptance of JSTOR's Terms and Conditions of Use, available at <http://www.jstor.org/page/info/about/policies/terms.jsp>. JSTOR's Terms and Conditions of Use provides, in part, that unless you have obtained prior permission, you may not download an entire issue of a journal or multiple copies of articles, and you may use content in the JSTOR archive only for your personal, non-commercial use.

Please contact the publisher regarding any further use of this work. Publisher contact information may be obtained at <http://www.jstor.org/action/showPublisher?publisherCode=rand>.

Each copy of any part of a JSTOR transmission must contain the same copyright notice that appears on the screen or printed page of such transmission.

JSTOR is a not-for-profit organization founded in 1995 to build trusted digital archives for scholarship. We work with the scholarly community to preserve their work and the materials they rely upon, and to build a common research platform that promotes the discovery and use of these resources. For more information about JSTOR, please contact [support@jstor.org](mailto:support@jstor.org).



The RAND Corporation is collaborating with JSTOR to digitize, preserve and extend access to *The Bell Journal of Economics*.

<http://www.jstor.org>

# Monopolistic competition with outside goods

**Steven C. Salop**

Bureau of Economics  
Federal Trade Commission

*The Chamberlinian monopolistically competitive equilibrium has been explored and extended in a number of recent papers. These analyses have paid only cursory attention to the existence of an industry outside the Chamberlinian group. In this article I analyze a model of spatial competition in which a second commodity is explicitly treated. In this two-industry economy, a zero-profit equilibrium with symmetrically located firms may exhibit rather strange properties. First, demand curves are kinked, although firms make “Nash” conjectures. If equilibrium lies at the kink, the effects of parameter changes are perverse. In the short run, prices are rigid in the face of small cost changes. In the long run, increases in costs lower equilibrium prices. Increases in market size raise prices. The welfare properties are also perverse at a kinked equilibrium.*

## 1. Introduction

■ The Chamberlinian (1931) zero-profit monopolistically competitive equilibrium has been explored and extended in a number of recent papers. These analyses have focused on the monopolistically competitive industry and have paid only cursory attention to the existence of an industry outside the Chamberlinian group. In this paper, a model of spatial competition is analyzed in which a second commodity is explicitly treated.

In this two-industry economy, a zero-profit equilibrium with symmetrically located firms may exhibit rather strange properties. First, demand curves are kinked, even though firms make “Nash” conjectures. If equilibrium is a tangency solution away from the kink, the short- and long-run responses to parameter changes are conventional. However, if equilibrium lies at the kink, the effects of parameter changes are perverse. In the short run, prices are rigid in the face of small cost changes. In the long run, increases in costs lower equilibrium prices. Interpreting the cost increase as an excise tax, this result states that the incidence of the tax is negative. Increases in market size raise prices. The welfare prop-

---

This is a revised version of a survey paper entitled “Monopolistic Competition Reconstituted” (1976). I am indebted to Don Hester, Lew Johnson, Bob Mackay, Perry Quick, Steve Salant, Joe Stiglitz, Andy Weiss, the referee and editor, for helpful comments and to Mary Ann Henry for editing and typing. The remarks in this paper represent only my personal views. They are not intended to be, and should not be construed as, representative of the views of any other member of the Federal Trade Commission staff or the individual Commissioners.

erties are also perverse at a kinked equilibrium. Decreases in cost and increases in market size lower both consumer and aggregate welfare.

In the next section the formal model is presented and the symmetric zero-profit Nash equilibrium (SZPE) is defined. Conditions for existence of the SZPE are derived in Section 3. Comparative statics and welfare properties are explored in Sections 4 and 5, respectively. The paper concludes with a short discussion of the deterrence equilibrium concept.

## 2. The basic model

■ In this section we analyze a variant of the traditional Hotelling (1929) model of spatial competition which is derived from Lerner and Singer (1937). In this variant the economy that is envisioned consists of two industries. The one upon which we focus is monopolistically competitive with differentiated brands and decreasing average costs; the other is a competitive industry producing a homogeneous commodity. Each of  $L$  consumers purchases either one unit or none<sup>1</sup> of the differentiated commodity according to preferences, prices, and the distribution of brands in product space. Remaining income is spent on the homogeneous commodity.

Each consumer has a most-preferred brand specification  $l^*$ . A brand  $l$  different from the most preferred specification is valued lower according to preferences in product space  $U(l, l^*)$ . The product space of the industry is taken to be an infinite line or the unit-circumference of a circle. While neither assumption is realistic, both allow the “corner” difficulties of the original Hotelling model to be ignored and an industry equilibrium with identical prices by equally-spaced firms to obtain. Eliminating the technical difficulties makes it simpler to analyze the qualitative equilibrium properties of the model. Thus, the model is a benchmark for subsequent analyses with nonuniform preferences across empirically validated product spaces. By eliminating technical problems, this model allows a focus on the essential interactions of firms in an industry.

If there are  $n$  brands of the differentiated commodity available at prices  $p_i$  and locations  $l_i$ , a consumer whose most preferred specification is  $l^*$  will purchase one unit from some brand if the maximum surplus of utility less price across brands outweighs the surplus from the homogeneous other good. Denoting that surplus by  $\bar{s}$ , we have the decision rule: Purchase one unit of the brand satisfying

$$\max_i [U(l_i, l^*) - p_i] \geq \bar{s}. \quad (1)$$

The traditional model of constant transport costs is captured with preferences given by

$$U(l_i, l^*) = u - c |l_i - l^*|, \quad (2)$$

where the “distance”  $|l_i - l^*|$  refers to the shortest arc length between  $l_i$  and  $l^*$ . In this case, equation (1) may be rewritten as follows:

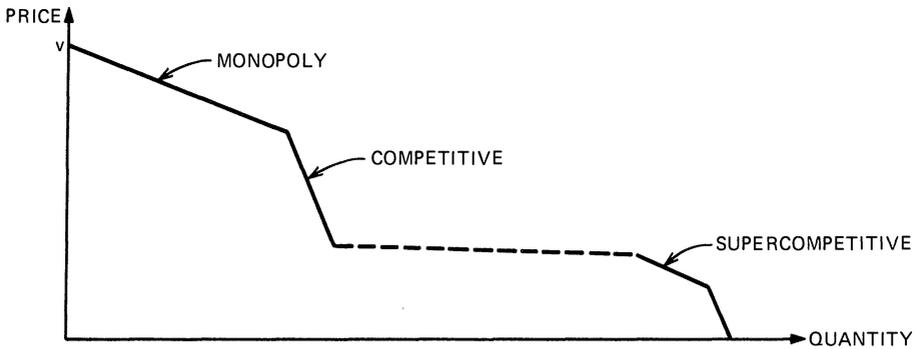
$$\max_i [v - c |l_i - l^*| - p_i] \geq 0, \quad (3)$$

where the effective reservation price is given by

$$v = u - \bar{s} > 0. \quad (4)$$

<sup>1</sup> The model easily generalizes to elastic demands.

FIGURE 1  
TYPICAL DEMAND CURVE



We now explore the existence and properties of a symmetric zero-profit Nash equilibrium (SZPE). By symmetric we mean an equilibrium in which the brands are equally spaced around the circular product space and charge identical prices. By zero profits, we mean an equilibrium in which free entry leads to a situation of each brand earning zero profits. The equilibrium is Nash in that each brand chooses a best price, given a perception that all other brands hold their prices constant. The conclusion discusses the requirement that the number of brands be integer-valued.

The methodology here consists of deriving the perceived demand curve for a single representative brand as a function of other brands' prices and locations and then finding a tangency between that demand curve and the average cost curve. Three regions of the representative brand's demand curve may be distinguished: the "monopoly," "competitive," and "supercompetitive" regions. The "monopoly" region consists of those prices in which the brand's entire market consists of consumers for whom the surplus of no other brand exceeds the surplus of the homogeneous outside good. The "competitive" region is composed of those prices in which customers are attracted who would otherwise purchase some other differentiated brand. The "supercompetitive" region consists of those prices in which all the customers of the closest neighboring brand are captured. These three regions of a typical demand curve are illustrated in Figure 1.

Suppose the representative brand charges price  $p$  and its nearest competitors located at distance  $1/n$  charge  $\bar{p}$ , as shown in Figure 2. We derive the regions of the demand curve as follows. In the absence of competition from other differentiated brands, the representative brand captures all consumers living within a distance where the net surplus given in (3) is nonnegative. Denoting the maximum distance by  $\hat{x}$  and substituting into (3) we have,

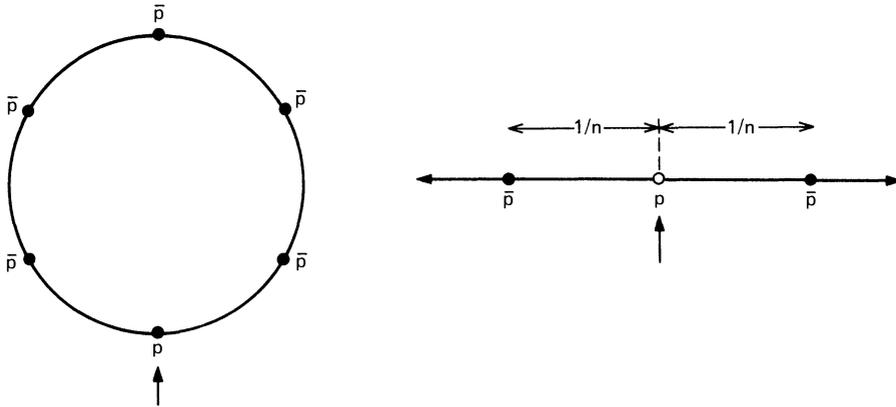
$$\hat{x} = \frac{v - p}{c} . \tag{5}$$

If there are  $L$  consumers around the circle, since the brand captures customers within a distance  $\hat{x}$  on each side, its monopoly demand  $q^m$  is given by

$$q^m = \frac{2L}{c} (v - p). \tag{6}$$

This defines the potential monopoly market of the representative brand.

FIGURE 2  
THE CIRCULAR MARKET



Those consumers residing in the potential monopoly market of two brands purchase from the one offering higher net surplus. If the brands are located a distance apart of  $1/n$  and the neighboring brand on one side charges a price  $\bar{p}$ , then from (3) the representative brand captures all those consumers within a distance  $x$  given by

$$v - cx - p \leq v - c\left(\frac{1}{n} - x\right) - \bar{p}. \tag{7}$$

Denoting by  $\bar{x}$ , the value for which (7) holds with equality, we have

$$\bar{x} = \frac{1}{2c} (\bar{p} + c/n - p) \tag{8}$$

and hence a firm's competitive demand  $q^c = 2L\bar{x}$  is given by

$$q^c = \frac{L}{c} (\bar{p} + c/n - p). \tag{9}$$

Differentiating (6) and (9), the slopes  $sl(D)$  of the demand curve in these two regions are given by

$$sl(D^m) = -c/2L \tag{10}$$

$$sl(D^c) = -c/L. \tag{11}$$

Thus, we have the unusual result that demand is more elastic in the monopoly region than in the competitive region. Moreover, as illustrated in Figure 1, the monopoly region comes at higher prices.

The two regions fit together as follows. Suppose the right-side neighbor has a potential monopoly market illustrated in Figure 3. At prices above  $v$ , the representative brand obtains no customers. As it begins lowering prices below  $v$ , it captures demand from the homogeneous good according to the monopoly

FIGURE 3  
MARKET SEGMENTS

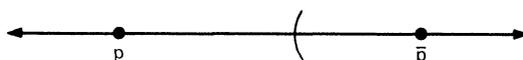


FIGURE 4  
MARKET OVERLAP



slope  $c/2L$ . Eventually its price becomes low enough that its monopoly market overlaps the monopoly market of its neighbor as illustrated in Figure 4. Now as it lowers price further, it begins to capture customers from its neighbor according to the steeper competitive slope  $c/L$ . At the kink in Figure 2, the monopoly regions just touch. Note that the kink arises here from the existence of the other industry, not from the non-Nash perceptions discussed by Sweezy (1939). It generalizes to higher dimensional spaces.<sup>2</sup>

At some lower price, even those customers residing at the neighbor are indifferent between the representative firm at  $p$  and the neighbor at  $\bar{p}$  (and additional surplus  $c(1/n)$ ). This price  $p_z$  is given by

$$p_z = \bar{p} - c/n. \quad (12)$$

At prices below  $p_z$ , the representative firm captures the entire market of its neighbor, for not only are those consumers residing at the neighbor willing to incur the surplus loss  $c/n$  for the price differential  $\bar{p} - p$ , but so are all the customers of the neighbor. Thus the representative brand's demand has a discontinuity at  $p_z$  from this "predatory" pricing.

The demand curve in Figure 2 displays the typical shape of these three regions. It shifts according to the prices and locations of the neighboring brands. Since demand can never exceed the monopoly demand, the kink always lies on that monopoly curve, as illustrated in Figure 5 (with supercompetitive regions deleted). Note that the market may be so competitive as to make the kink nonexistent. This occurs when the neighbors' potential monopoly market includes the location of the representative brand.

### 3. Existence of a symmetric zero profit equilibrium (SZPE)

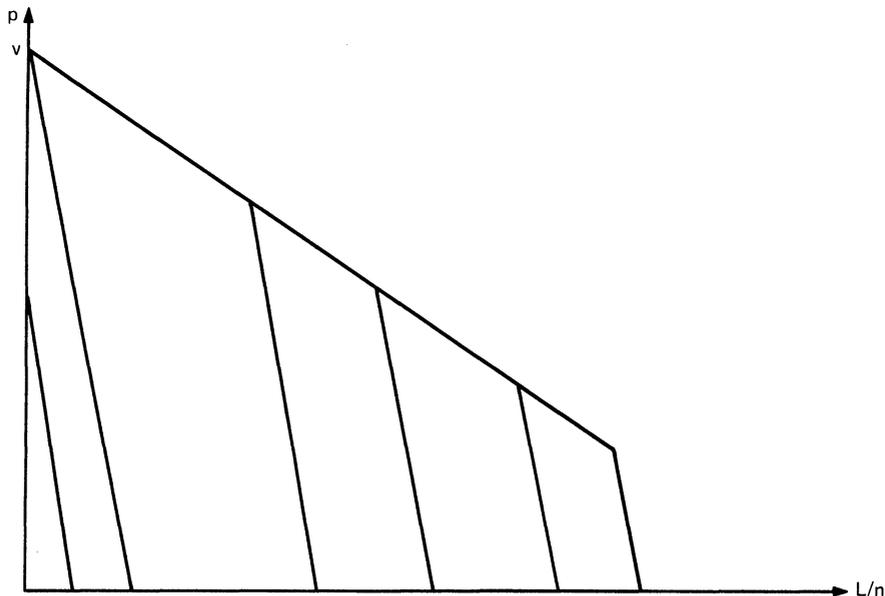
■ A SZPE is defined as a price  $p$  and a number of brands  $n$  such that every equally spaced<sup>3</sup> Nash price setter's maximum profit price choice earns zero profits. We ignore the additional requirement that the number of brands must be an integer and discuss it later. In addition, the potential nonexistence of equilibrium arising from the discontinuity in demand is also postponed. If an equilibrium exists, the representative brand's demand curve and average cost curve will be tangent, for then the zero-profit point is surely also one of maximum profits. Three equilibrium configurations are possible, as illustrated in Figure 6, where the monopoly, kinked, and competitive equilibrium prices are denoted by subscripts ( $m, k, c$ ), respectively.

At the monopoly equilibrium, some consumers lying between two neighboring brands may not purchase the differentiated commodity. Thus, the markets

<sup>2</sup> This may be confirmed in a two-dimensional product space. The monopoly market is circular, while competitive markets are polygonal.

<sup>3</sup> This equilibrium concept is static. In a dynamic context, it assumes that firms may costlessly relocate in response to entry and, in fact, do relocate. Thus, equal spacing is maintained. For a discussion of an alternative equilibrium concept, see the conclusions.

FIGURE 5  
FAMILY OF DEMAND CURVES



of neighbors may not overlap and each can act as a monopolist, constrained only by the outside commodity. Monopoly equilibria with and without overlap are pictured in Figure 6. At a kinked equilibrium, markets just touch. As illustrated in Figure 6, since the extension of the monopoly demand curve lies above the average cost curve, the monopoly price  $p_m$  lies below the kinked equilibrium price  $p_k$ . At the competitive equilibrium configuration, monopoly markets completely overlap. However,  $p_c$  may be above or below  $p_m$ , depending on demand and technologies.

It is easy to show graphically which equilibrium configuration obtains for any set of technology and demand parameters  $\{F, m, v, c, L\}$ . Simply drawing the average cost curve and the entire family of demand curves, existence of an equilibrium configuration requires maximum profits equal to zero-point  $E$  in Figure 7. A zero-profit point like  $G$  does not satisfy maximum profits because it is dominated by a point like  $F$ .

FIGURE 6  
EQUILIBRIUM CONFIGURATIONS

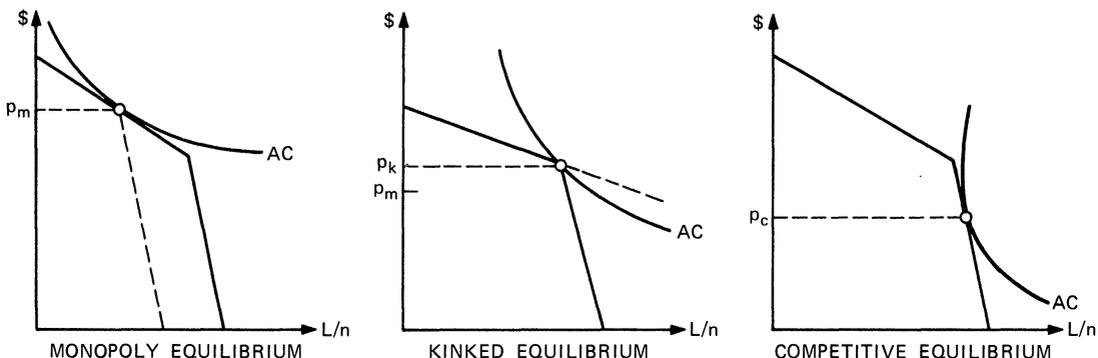
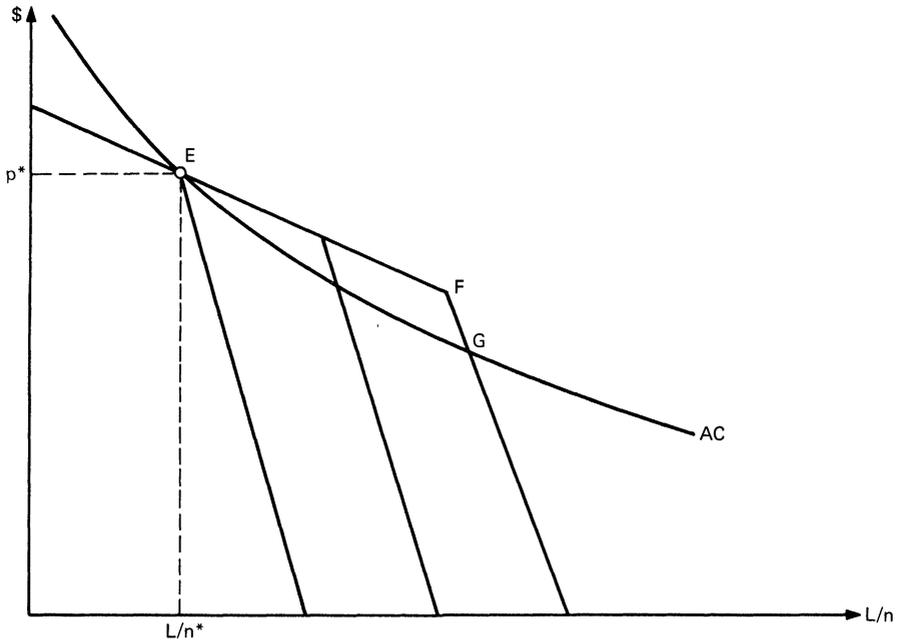


FIGURE 7  
EXISTENCE OF EQUILIBRIUM



The SZPE satisfies two conditions: marginal revenue (less than or) equal to marginal cost and price equal to average cost. For constant marginal cost  $m$  and fixed cost  $F$ , the SZPE is given by

$$p + q \frac{dp}{dq} \leq m \tag{13}$$

$$p = m + F/q, \tag{14}$$

and from symmetry, if the equilibrium has no gaps,

$$q = L/n. \tag{15}$$

At the monopoly equilibrium  $dp/dq$  is given by  $sl(D^m)$  in (10); at the competitive equilibrium by  $sl(D^c)$  in (11); and at the kinked equilibrium by a slope between  $sl(D^m)$  and  $sl(D^c)$ . Substituting (15) and (10) into (13) and (14), the monopoly price and number of brands<sup>4</sup> are given by

$$p_m = m + c/2n_m \tag{16}$$

$$n_m = \frac{1}{\sqrt{2}} \sqrt{cL/F}. \tag{17}$$

Using (11) instead of (10), the competitive equilibrium is given by<sup>5</sup>

$$p_c = m + c/n_c \tag{18}$$

$$n_c = \sqrt{cL/F}. \tag{19}$$

<sup>4</sup> There may be gaps at a monopoly equilibrium. This calculation yields the maximum number of brands at a monopoly equilibrium.

<sup>5</sup> See Grubel (1963) for a short derivation of the competitive equilibrium.

The values  $(p_k, n_k)$  for a kinked equilibrium lie between the values given in (16)–(19). Since there is no tangency at a kinked equilibrium, (13) holds as an inequality. Instead of being given by the equality in (13), price is given by the monopolistic demand function, or

$$p_k = v - (c/L)q = v - c/n. \quad (20)$$

Solving (20) and the price equal to average cost condition given by equation (14) for equilibrium variety  $n_k$ , we have

$$\frac{F}{L} n_k + c/n_k = v - m. \quad (21)$$

The monopoly equilibrium configuration requires the very restrictive condition that the exogenously given average cost curve be tangent to the exogenously given demand curve or  $v - m = \sqrt{2cF/L}$ .<sup>6</sup> We ignore this limiting case for the remainder of the analysis. The competitive equilibrium configuration occurs for all parameter values such that  $v - m \geq \frac{3}{2}\sqrt{cF/L}$ .<sup>7</sup> The kinked equilibrium configuration occurs for values of  $v - m$  in the interval  $[\sqrt{2cF/L}, \frac{3}{2}\sqrt{cF/L}]$ , which is small relative to the range of values  $v - m$  can assume.<sup>8</sup>

The demand discontinuity can imply the nonexistence of any SZPE.<sup>9</sup> Recalling from (12) that the representative firm can capture its neighbor's entire market at prices below  $\bar{p} - c/n$ , an additional condition for existence of a SZPE is that such pricing behavior is unprofitable. A sufficient condition for this is that the predatory price  $\bar{p} - c/n$  does not exceed marginal cost  $m$ , for price equal to or below marginal cost necessarily is a losing strategy in the presence of fixed costs. Referring to (16) and (18), supercompetitive behavior is not profitable, since the equilibrium price is no greater than  $m + c/n$ . Similarly, if marginal costs are increasing, as with  $U$ -shaped  $AC$  curves, then the market-capturing price lies below the minimum  $AC$  price. However, if marginal costs are decreasing, then such price cuts may be profitable and cause nonexistence of a SZPE.

#### 4. Comparative statics

■ As the exogenous technological or demand parameters  $\{F, m, v, c, L\}$  vary, the equilibrium price-variety pair also changes. These changes may be calculated from the equilibrium values in equations (16)–(19).

□ **Competitive equilibria.** The comparative statics at competitive equilibria are straightforward and traditional. Substituting (19) into (18) we have

$$p_c = m + \sqrt{\frac{cF}{L}} \quad (22)$$

$$n_c = \sqrt{\frac{cL}{F}}. \quad (23)$$

<sup>6</sup> For the derivation, see the derivation of equation (31) below when profits are zero.

<sup>7</sup> The derivation is as follows. Referring to Figure 6, the equilibrium  $p_c$  derived in (18) and (19) must lie below the monopoly portion of the demand curve, or  $v - \frac{1}{2}(c/L)(L/n_c) \geq p_c$ . Substituting for  $p_c$  and  $n_c$ , the stated condition obtains.

<sup>8</sup> This interval may be larger for alternative technological and demand specifications.

<sup>9</sup> See Roberts-Sonnenschein (1977) for examples of discontinuous reaction functions leading to nonexistence of equilibrium.

As fixed costs ( $F$ ) rise, price rises and equilibrium variety falls. Changes in marginal costs ( $m$ ) are fully shifted onto consumers; equilibrium variety remains the same. Surprisingly perhaps, changes in the net valuation  $v$  have no effect on the equilibrium. The market is competitive enough that aggregate demand is unaffected by changes in this demand price.

As the value of product differentiation ( $c$ ) falls, prices fall and variety falls. As market size ( $L$ ) rises, prices fall and variety rises. As  $c/L$  decreases, demand becomes more elastic and price moves toward marginal cost. Thus, the ratio  $c/L$  is the relevant measure of monopolistic product differentiation in the model. For  $U$ -shaped average costs, perfect competition obtains when  $c/L = 0$ , for then every brand faces a perfectly elastic demand function.

□ **Kinked equilibria.** The comparative statics at kinked equilibria are all perverse. An increase in either fixed or marginal costs lowers prices. This is illustrated diagrammatically below as a movement from  $E$  to  $E'$ . Intuitively, cost increases reduce the equilibrium number of brands, allowing the remaining brands to further exploit scale economies. This is a very striking result. If the increase in costs is interpreted as an excise tax levied on the industry, then the incidence of that excise tax is negative at the kinked equilibrium. In terms of consumer welfare, the lower price is offset by the decline in variety, of course. However, it is shown in the next section that consumer welfare does rise from the tax, even if the proceeds of the tax are ignored.

It should be emphasized that this perverse reaction to a cost increase is a long-run response that results from the exit of marginal firms. In the short run, there is no reaction at all. Since the marginal revenue curve is discontinuous at the kinked equilibrium, a small change in marginal costs induces no price response. Thus, the industry responds to a small marginal cost increase as follows. In the short run, prices and quantities do not change, though profits fall below normal (zero). These losses induce some firms to exit, resulting in higher demand for those that remain. This increased demand allows the remaining firms to better exploit scale economies, resulting in decreased long-run prices.

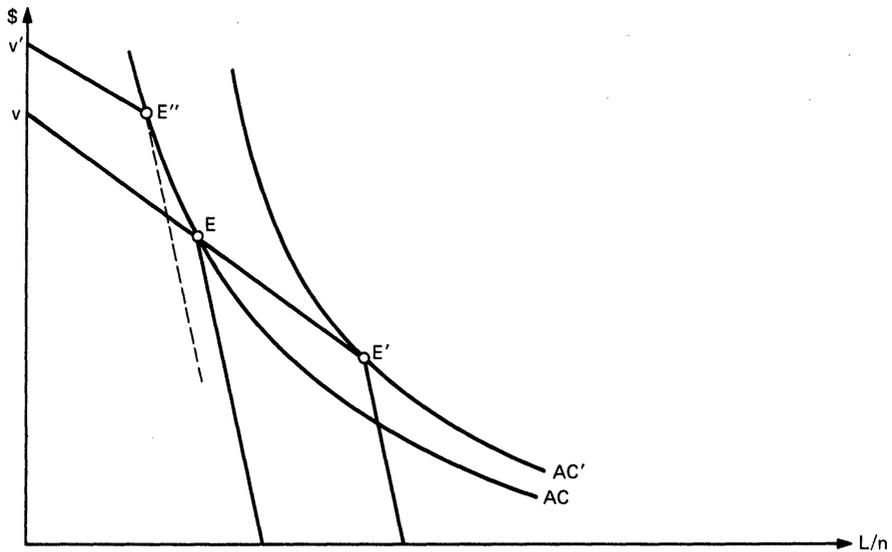
An increase in the valuation ( $v$ ) raises price and variety, as illustrated by the movement from  $E$  to  $E''$ . As may be seen from Figure 8, price rises by more than the increase in valuation, as scale economies are lost. As with cost increases, the welfare effect of this increase in valuation is also perverse; it may be shown that consumer welfare falls. Interpreting the increase in valuation as arising from informative advertising and the cost increase as the cost of that informative advertising, the valuation effect lowers welfare, while the cost effect raises welfare.

Decreases in  $c/L$ , arising from either an increase in market size ( $L$ ) or a decrease in the value of product differentiation ( $c$ ), raise prices in equilibrium, as illustrated in Figure 9 as a movement from  $E$  to  $E'$ . As with the other comparative statics discussed, this result is the reverse of what occurs in the competitive equilibrium configuration.

## 5. Welfare analysis

■ **Product selection.** It has been pointed out by Spence (1976) and others that the production of some unprofitable commodities may be optimal and the production of some profitable commodities may be nonoptimal. For the circular

FIGURE 8  
COMPARATIVE STATICS OF  $(v, m, F)$ : KINKED EQUILIBRIUM

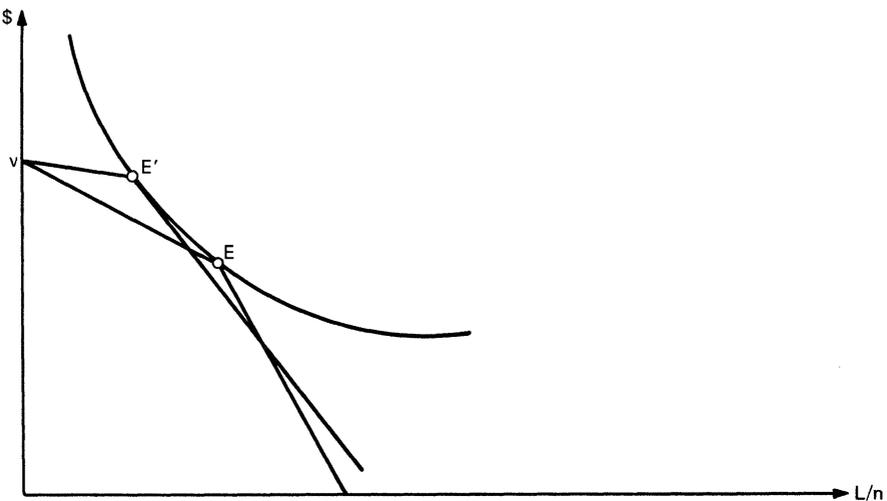


market, this may be tested by comparing the condition under which a segment of the market will be served by a monopolist (the profitability condition) with the condition under which service yields positive net surplus (the optimality condition).

Suppose each brand produces up to the point where the net benefit to the marginal consumer, who is at a distance  $x^*$  from the brand serving him, is zero. Then, the net social benefit per brand of serving the entire circular market is:

$$B = 2L \int_0^{x^*} (v - cx - m)dx - F, \quad (24)$$

FIGURE 9  
COMPARATIVE STATICS OF  $c/L$ : KINKED EQUILIBRIUM



where the marginal consumer's benefit is given by

$$v - cx^* - m = 0. \quad (25)$$

Substituting (25) into (24) and integrating, we have

$$B = \frac{L}{c} (v - m)^2 - F, \quad (26)$$

and this surplus is nonnegative ( $B \geq 0$ ) if and only if the following optimality condition is satisfied:

$$v - m \geq \sqrt{\frac{cF}{L}}. \quad (27)$$

On the other hand, a monopolistic firm will choose to serve any segment of the market only if its profits are nonnegative,<sup>10</sup> where profits for the monopoly portion of the demand function are given by

$$\Pi_m = \left( v - \frac{c}{2L}q - m \right)q - F. \quad (28)$$

Maximizing (28) with respect to  $q$ , we have

$$q_m = \frac{L}{c} (v - m). \quad (29)$$

Substituting (29) into (28), profits are given by

$$\Pi_m = \frac{L}{2c} (v - m)^2 - F. \quad (30)$$

Profits are nonnegative ( $\Pi_m \geq 0$ ) if and only if the following profitability condition is satisfied:

$$v - m \geq \sqrt{\frac{2cF}{L}}. \quad (31)$$

Comparing the optimality and profitability conditions, we see that profitability is sufficient but not necessary for optimality. All markets served should be served, but not *vice versa*.

□ **Optimal vs. equilibrium variety.** Given that the entire circular market should be served, the optimal price-variety pair may be compared with the equilibrium price-variety pair. A tradeoff between price and variety exists because of the scale economies present in production.

If  $n$  firms operate and serve the entire unit-circumference market, then the marginal consumer travels a distance  $\frac{1}{2}n$  and a consumer located at  $x \leq \frac{1}{2}n$  obtains a surplus in excess of marginal cost of  $v - m - cx$ . Since there are  $L$  consumers per unit distance and  $2n$  intervals of length  $\frac{1}{2}n$ , total surplus is given by

$$W = 2n \int_0^{1/2n} (v - m - cx)Ldx - nF. \quad (32)$$

<sup>10</sup> It should be noted that if one monopolist does not wish to serve one segment, no monopolist will serve any segment since the circular market is symmetric.

Integrating, we have:

$$W = \left( v - m - \frac{1}{4} \frac{c}{n} \right) L - nF. \quad (33)$$

The interpretation of (33) is the following. Since (i) the marginal consumer travels a distance of  $\frac{1}{2}n$  in product space, (ii) the consumer who travels the shortest distance (zero) obtains his most preferred brand, and (iii)  $L$  consumers are distributed uniformly in product space, the average distance travelled is  $\frac{1}{4}n$  at an imputed cost of  $c$  per unit. Then the average net surplus per consumer is  $v - m - c/4n$ . Total fixed costs are  $nF$ . Maximizing (33) with respect to the number of brands  $n$  to find the optimal price-variety mix, we have

$$n^* = \frac{1}{2} \sqrt{\frac{cL}{F}}. \quad (34)$$

Comparing this optimum to the possible equilibria given by (17), (19), and (21) we have

$$n^* < n_m < n_k < n_c. \quad (35)$$

That is, optimal variety is less than equilibrium variety for this circular market, if the market should be served.

This result of too many brands is not robust, but rather depends crucially on the distribution of consumers and preferences. As Spence (1976) and others<sup>11</sup> have pointed out, the optimum depends on the difference between the average surplus and the surplus of the marginal consumer relative to fixed costs; the value of adding an extra brand (and respacing the others) effectively converts marginal consumers to average ones, at fixed cost  $F$ .<sup>12</sup>

Graphically, the comparison of the equilibrium with the optimum may be made as follows. The planning problem in (33) is equivalent to maximizing average consumer welfare minus price  $W(n,p)$  subject to the price equal to average cost breakeven constraint.<sup>13</sup> Since the average consumer travels a distance  $\frac{1}{4}n$  in product space, we have

$$\max W(n,p) = v - p - \frac{1}{4} \frac{c}{n} \quad (36)$$

$$\text{subject to } p = m + \frac{F}{L} n. \quad (37)$$

Equation (36) defines linear indifference curves in  $(p, L/n)$  space with slope of  $-\frac{1}{4}(c/L)$  while (37) expresses the constraint. As illustrated in Figure 10, a smaller value of  $S$  expresses a higher surplus  $v - S$ . Then, the optimum lies at the point where the average cost has slope equal to  $-\frac{1}{4}(c/L)$ , whereas equilibrium lies at the point where the average cost curve has slope between  $-\frac{1}{2}(c/L)$  (for monopoly equilibrium) and  $-c/L$  (for competitive equilibrium).

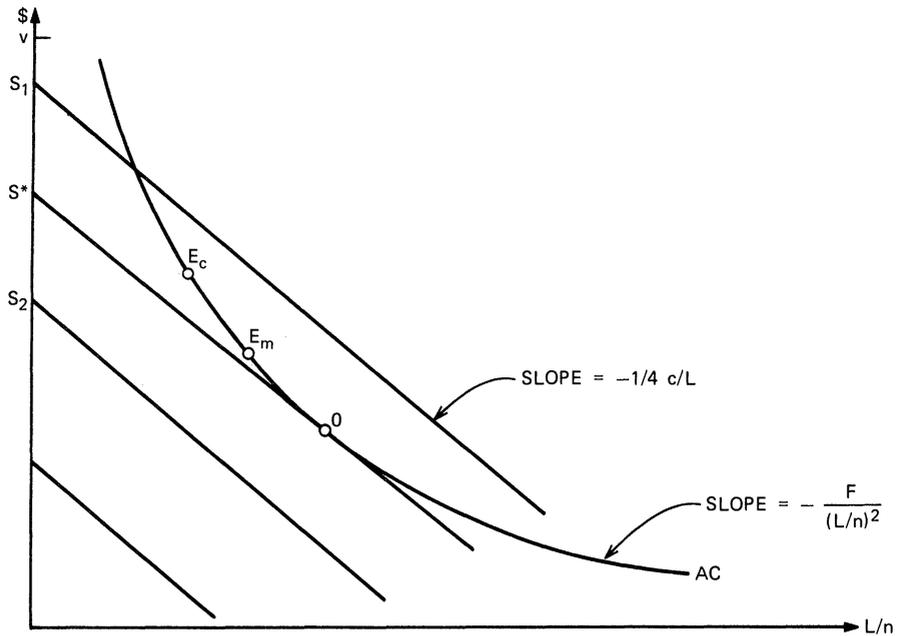
A graphical representation of the optimum vs. equilibrium price-variety pair may be used to show the welfare effects of the comparative statics at

<sup>11</sup> For example, Dixit and Stiglitz (1977) and Lancaster (1975).

<sup>12</sup> It can be shown that any utility function that is concave in distance will yield excess variety, for a uniform consumer distribution. Convex functions are necessary for deficient variety.

<sup>13</sup> Since consumers have inelastic demands in this example, that price does not equal marginal cost introduces no distortion.

FIGURE 10  
EQUILIBRIUM VS. OPTIMAL VARIETY



the kinked equilibrium. (See Figure 11.) For example, an increase in costs from  $AC$  to  $AC'$  that lowers prices will improve welfare, since the slope of the indifference curve ( $-1/4c/L$ ) is flatter than the slope of the monopoly demand curve ( $-1/2c/L$ ). Thus, movements down the demand curve represent higher welfare as illustrated below by comparing  $S$  to  $S'$ . Similar analysis will show that

FIGURE 11  
DERIVATION OF OPTIMAL VARIETY

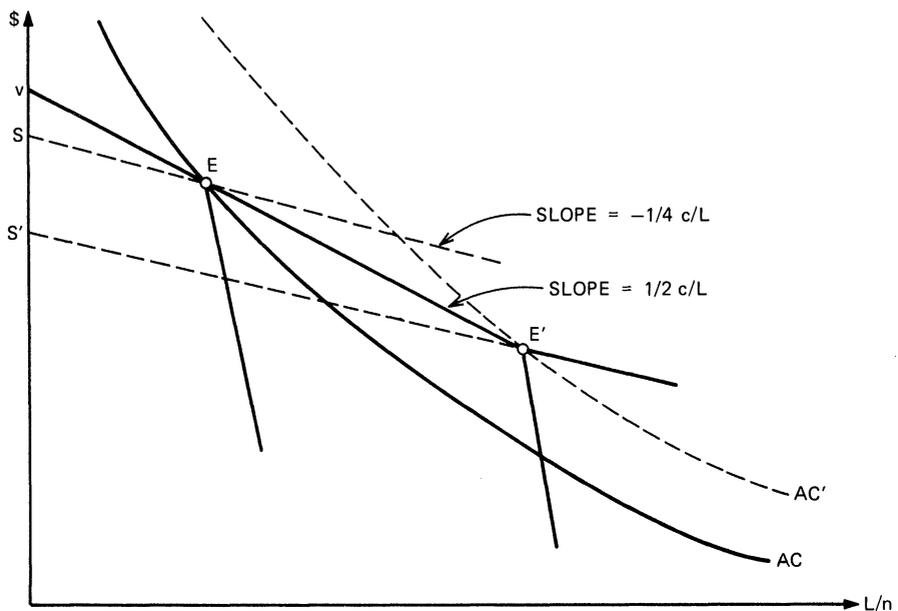
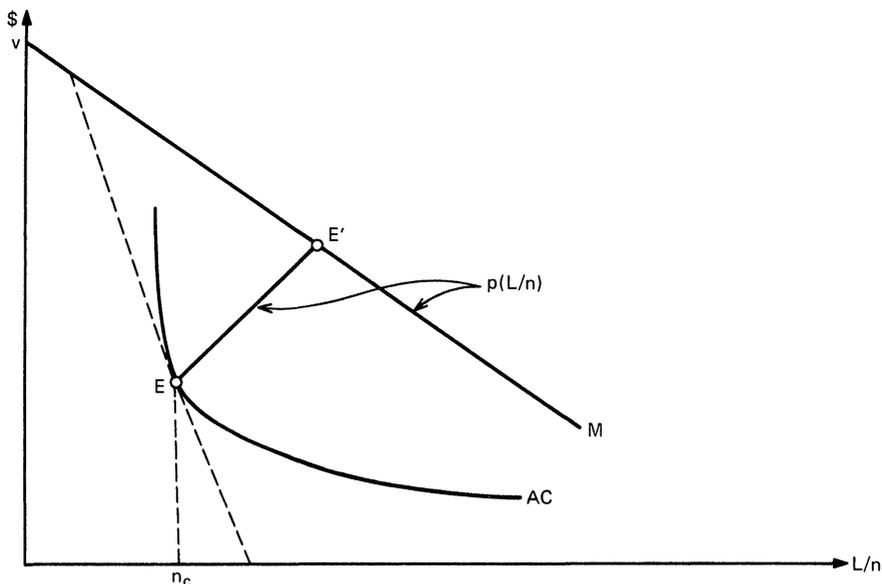


FIGURE 12  
DERIVATION OF THE  $p(L/n)$  FUNCTION



an increase in valuation ( $v$ ) and an increase in market size ( $L$ ) lowers consumer welfare.

Finally, we can determine the optimal number of brands given monopolistically competitive pricing. Spence (1976) shows that for his partial equilibrium model, the market solution is optimal. For the circular model studied here, the optimum is either the market equilibrium or complete monopoly. We may prove this as follows. First we derive the Nash equilibrium price for different (exogenously given) numbers of symmetrically spaced brands. We denote this relationship by  $p(L/n)$ .<sup>14</sup> The  $p(L/n)$  function is illustrated in Figure 12 as  $EE'M$ , where we assume the SZPE (point  $E$ ) is competitive. The complete monopoly equilibrium is labeled  $M$ , and  $E'M$  is a portion of the monopoly demand curve.

The consumer welfare maximum could then be found by placing indifference curves, which have slope  $-1/4(c/L)$ , in Figure 12. It is clear that the optimum must lie at  $E$  or  $M$ . If the SZPE were kinked, say at  $E'$ , then  $p(L/n)$  would be the portion  $E'M$ , and the optimum would lie at the complete monopoly point. Thus, for the circular industry, optimal entry policy is either free entry or entry restricted to the point of each brand having a complete monopoly market.

## 6. Conclusions

■ In the example studied here, explicit attention has been paid to the role a second industry (outside goods) plays in determining the properties of monopolistically competitive equilibrium. This focus required us to ignore the possibility of

<sup>14</sup> The derivation of  $p(L/n)$  relies on the following observation. For any number of brands  $n$ , there exists a level of fixed costs  $F'$  such that an  $n$ -brand SZPE obtains (or the market is not served). Thus, an  $n$ -brand Nash equilibrium, when fixed costs are  $F$ , yields equilibrium price  $p(L/n)$  and excess profits per brand of  $\Pi = F' - F$ .

nonuniform preferences across more complicated and realistic product spaces or different technologies across brands.

The major contribution of the approach taken here, which was first noted by Lerner and Singer (1937), is to provide a rationalization of the kinked demand curve in terms of symmetric "Nash" conjectural variations. Previous rationalizations by Sweezy (1939) and others were based upon asymmetric competitive responses to price increases and decreases. This paper goes beyond Lerner and Singer by deriving the industry equilibrium and analyzing its properties.

The industry equilibrium model displays conventional properties when equilibrium occurs at a Chamberlinian tangency away from the kink. On the other hand, the properties of equilibria occurring at the kink are perverse. In the short run, industry prices do not adjust to small cost changes, as noted by Sweezy in his model. It is only through the process of entry and exit that the industry adjusts to cost changes. Moreover, at the kinked equilibrium, the long-run response to a cost increase is exit by some brands followed by a decrease in industry prices, as remaining brands better exploit scale economies.

The short-run price rigidity of the kinked equilibrium accords with casual empiricism. However, the long-run properties are more difficult to confirm or reject, since they depend on longer run entry adjustments. Moreover, the actual symmetric example analyzed entails the abstract and unrealistic assumptions of uniform preferences around a circular product space and identical cost functions and valuations among competing brands. While these assumptions considerably simplify the theoretical analysis, they make empirical confirmation more difficult.

Other shortcomings of the approach taken here are the zero-profit and costless relocation assumptions we have made. Because the technology is characterized by an indivisible fixed cost, the number of brands must be integer-valued. Therefore, free entry need not lead to a zero-profit equilibrium, as originally pointed out by Kaldor (1935) and analyzed by Eaton (1976). An interesting "deterrence" equilibrium concept built on these foundations has been explored for a circular market by Hay (1976) and Schmalensee (1977) and for other spatial markets by Prescott and Visscher (1977). In a deterrence equilibrium sequential entrants locate in such a way that no new entrant wishes to locate in the interval between two firms. As a result, the deterrence equilibrium has half the number of brands as the SZPE.

In the model analyzed here, the deterrence equilibrium configuration is generally that point in the  $p(L/n)$  curve with the number of brands  $n$  equal to  $\frac{1}{2}$  the number at the SZPE.<sup>15</sup> If the SZPE is competitive, the deterrence equilibrium may be competitive, kinked, or at the monopoly point; which equilibrium occurs depends on the particular parameter values. Hay showed that if the deterrence equilibrium is competitive, prices are higher than at the SZPE. However, kinked or monopoly deterrence equilibria may result in lower prices than the competitive SZPE. Finally, if the SZPE is kinked, the deterrence equilibrium lies at the monopoly point and entails lower prices and unserved segments of the circular market.

---

<sup>15</sup> The exception arises when this point on  $p(L/n)$  lies below the monopoly point on the monopoly portion of the demand curve. In this case, while the number of brands is still halved, the remaining brands raise price and lower output to the monopoly point. Hence, the deterrence equilibrium entails unserved segments of the circular market.

In closing, it should be reemphasized that the exact results derived here merely reflect the example used. That there is excess variety at equilibrium is not robust. The kink appears robust as the number of dimensions of product space increases. However, as Archibald-Rosenbluth (1975) and Weiss (1977) point out, equilibrium may not exist with higher dimensional product spaces.

## References

- ARCHIBALD, G.C. AND ROSENBLUTH, G. "The 'New' Theory of Consumer Demand and Monopolistic Competition." *Quarterly Journal of Economics*, Vol. 89 (1975).
- CHAMBERLIN, E.H. *The Theory of Monopolistic Competition*. Cambridge, Mass.: Harvard University Press, 1931.
- DIXIT, A.K. AND STIGLITZ, J.E. "Monopolistic Competition and Optimum Product Diversity." *The American Economic Review*, Vol. 67 (1977).
- EATON, B.C. "Free Entry in One-Dimensional Models: Pure Profits and Multiple Equilibria." *Journal of Regional Science* (1976).
- GRUBEL, H.G. "Wastes of Monopolistic Competition." (Based on a lecture by J. Tobin.) Yale University, 1963.
- HOTELLING, H. "Stability in Competition," *Economic Journal*, Vol. 39 (1929).
- KALDOR, N. "Market Imperfection and Excess Capacity." *Economica N.S.* 2 (1935).
- LANCASTER, K. "Socially Optimal Product Differentiation." *The American Economic Review*, Vol. 65 (1975).
- LERNER, A.P. AND SINGER, H.W. "Some Notes on Duopoly and Spatial Competition." *Journal of Political Economy*, Vol. 45 (1937).
- PRESCOTT, E. AND VISSCHER, M. "Sequential Location Decisions among Firms with Foresight." *The Bell Journal of Economics*, Vol. 8 (1977).
- ROBERTS, J. AND SONNENSCHN, H. "On the Foundations of the Theory of Monopolistic Competition." *Econometrica*, Vol. 75 (1977).
- SCHMALENSEE, R. "Entry Deterrence in the Ready-to-Eat Breakfast Cereal Industry." *The Bell Journal of Economics*, Vol. 9 (1978).
- SPENCE, A.M. "Product Selection, Fixed Costs and Monopolistic Competition." *Review of Economic Studies*, Vol. 43 (1976).
- SWEEZY, P.M. "Demand under Conditions of Oligopoly." *Journal of Political Economy*, Vol. 47 (1939).
- WEISS, A. "Spatial Competition with Two Dimensions." Unpublished manuscript, Bell Laboratories, 1977.