THE MIT Press

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Source: *The Quarterly Journal of Economics*, Vol. 94, No. 2 (Mar., 1980), pp. 429-436
Published by: The MIT Press
Stable URL: http://www.jstor.org/stable/1884551
Accessed: 31/08/2010 19:30

MARKET STRUCTURE AND INNOVATION: A REFORMULATION

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I

The relationship between market structure and innovative activity has attracted a greal deal of attention from economists over the last two decades. One of the most interesting recent additions to the literature has been provided by Glenn Loury [1979]. He analyzes "a world in which . . . firms compete for the constant, known, perpetual flow of rewards . . . that will become available only to the first firm that introduces [some given] innovation" [Loury, 1979, p. 397]. Following Kamien and Schwartz [1976], he assumes that individual firms face a stochastic relationship between investment in R & D and the time at which a usable innovation (a "new" technology) is produced. The interaction of firms competing to introduce the innovation is then modeled as a noncooperative game [Scherer, 1967]. Loury's major conclusions are as follows:

i. As the number of firms in the industry increases, the equilibrium level of firm investment in R & D declines.
ii. When there are initial increasing returns to scale in the R & D technology, then a zero expected profit industry equilibrium with a finite number of firms always involves "excess capacity" in the R & D technology.
iii. Given a fixed market structure, industry equilibrium will have each firm investing more in R & D than is socially optimal.
iv. When there are initial increasing returns to scale in the R & D technology, competitive entry leads to more than the socially optimal number of firms in the industry.

It turns out that conclusions (i) and (ii) are sensitive to Loury's specification of the costs of R & D. In this paper we investigate the effects of an alternative specification.

II

We begin with a development of Loury's basic model. Consider an industry with n firms, each competing to be the first to introduce a new technology. The reward to being first to introduce the new

technology is a fixed sum V.[1] Once any firm introduces the new technology, all other firms lose their investments in R & D.

Loury assumes that each firm's introduction time is a random variable τ distributed according to

$$Pr(\tau \leq t) = 1 - e^{-h(x)t},$$

where x is the firm's investment in R & D. Note that

$$E(\tau) = h(x)^{-1}$$

Thus, the firm experiences a constant instantaneous probability $h(x)$ that its research will produce a usable new technology.

Assume that h is twice differentiable, strictly increasing, and satisfies the conditions that

(i) $$h(0) = 0 = \lim_{x \to \infty} h'(x),$$

(ii) $$h''(x) \begin{array}{c} > \\ < \end{array} 0 \quad \text{as } x \begin{array}{c} < \\ > \end{array} \bar{x},$$

(iii) $$\frac{h(x)}{x} \begin{array}{c} > \\ < \end{array} h'(x) \quad \text{as } x \begin{array}{c} > \\ < \end{array} \tilde{x}.$$

This set of assumptions allows for the possibility of initial increasing returns to scale in R & D ($\bar{x}$ may be strictly positive).

Consider a particular firm i. By choosing an investment in R & D of x_i, the firm is in fact purchasing a random technology introduction time $\tau_i(x_i)$. However, the firm's rivals purchase random technology introduction times of their own. Let $\bar{\tau}_i$ be the random variable representing the date at which the first of them introduces the new technology. Then

$$\bar{\tau}_i = \min_{j \neq i} \{\tau_j(x_j)\}$$

and

$$Pr(\bar{\tau}_i \leq t) = 1 - Pr\ (\tau_j > t, \text{ for all } j \neq i)$$

$$= 1 - \exp\left\{\sum_{j \neq i} h(x_j)\right\}.$$

Define

$$a \equiv \sum_{j \neq i} h(x_j).$$

Then the expected benefit of investing in R & D is

$$EB = \int_0^\infty Pr(\hat{\tau}_i = t) \left\{ \int_0^t Pr(\tau = s)Ve^{-sr}ds \right\}dt$$

$$= \int_0^\infty ae^{-at} \left\{ \int_0^t he^{-hs}e^{-sr}ds \right\}dt$$

$$= \frac{Vh}{a + h + r}.$$

The model we develop departs from Loury's in our specification of the costs of R & D. Loury assumes that the random variable $\tau(x)$ is purchased by paying x at $t = 0$. We assume that $\tau(x)$ is purchased by paying a fixed cost of F and incurring a flow cost of x. The firm will continue to pay this flow cost until either it or one of the other firms in the market produces a usable new technology.[2] Hence, expected costs are

$$EC = \int_0^\infty \left\{ \int_0^t xe^{-rs}ds \right\}Pr(\hat{\tau}_i = t \text{ or } \tau_i = t)dt + F$$

$$= \int_0^\infty \left\{ \int_0^t xe^{-rs}ds \right\}(a + h)e^{-(a+h)t}dt + F$$

$$= \frac{x}{a + h + r} + F,$$

where F is the fixed cost and $x/(a + h + r)$ is the expected variable cost.

Expected profit is simply expected benefits minus expected costs. That is,

(1) $$E\pi = EB - EC$$

$$= \frac{Vh - x}{a + h + r} - F.$$

The objective of the firm is to maximize expected profit. The first-order condition associated with this maximization is

(2) $$\frac{\partial E\pi}{\partial x} = \frac{(a + r)(Vh' - 1) - (h - xh')}{(a + h + r)^2} = 0.$$

2. Since completing this paper we have become aware of a similar (but more general) approach due to Jennifer Reinganum. She allows firms to accumulate a stock of knowledge as they invest in R & D and assumes that the instantaneous probability of introducing a new technology is a function of both the current rate of investment and the accumulated stock of knowledge. The industry is then modeled as a differential game. This is a considerably more sophisticated model than ours, but apparently yields similar results.

Equation (2) implies that $V = (a + h + r - xh')/(a + r)h'$ at the optimal x. Hence from (1),

$$(3) \qquad E\pi = \frac{h - xh'}{(a + r)h'} - F.$$

Nonnegative profits require at least that $h > \hat{x}h'$. This is exactly opposite of Loury's result (see (ii) above).

Consider next the effect of changes in a on the optimal x (call it $\hat{x}$):

$$\frac{d\hat{x}}{da} = \frac{-(Vh' - 1)}{[(a + r)V - x]h''}.$$

Nonnegative profits implies that $h > \hat{x}h'$. But this means that $h'' < 0$ at $\hat{x}$ (allowing for entry and exit).[3] The first-order condition for $\hat{x}$ also implies that $(Vh' - 1)$ has the same sign as $h - \hat{x}h'$. Thus, $d\hat{x}/da > 0$ as long as the firm earns nonnegative profits. If one interprets a as the degree of rivalry, the implication of the above result is that greater rivalry stimulates R & D activity. This is another conclusion we obtain that differs from Loury's.

The next step is to analyze the full industry equilibrium. Following Loury, we shall focus on the symmetric Nash equilibrium. In this case $a = (n - 1)h(\hat{x})$. Let the implicit solution to $\partial E\pi/\partial x = 0$ be denoted $\hat{x} = H(a)$. Then $\hat{x} = H[(n - 1)h(\hat{x})]$ in equilibrium. Hence,

$$(4) \qquad \frac{d\hat{x}}{dn} = \frac{H}{\partial a} h \bigg/ 1 - \left(\frac{\partial H}{\partial a}\right)(n - 1)h'.$$

Since $\partial H/\partial a = d\hat{x}/da > 0$, $d\hat{x}/dn$ has the same sign as $1 - (\partial H/\partial a)(n - 1)h'$. As a "stability" condition analogous to Loury's (but not identical), assume throughout the rest of this note that $1 - (\partial H/\partial a)(n - 1)h' > 0$ so that $d\hat{x}/dn > 0$. Summarizing, we have

THEOREM 1. Assume that $1 - (\partial H/\partial a)(n - 1)h' > 0$. Then as the number of firms in the industry increases, the equilibrium investment in R & D increases.[4]

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3. That $h''(\hat{x})$ is negative can also be derived from the second-order condition associated with $\hat{x}$. From (2),

$$\frac{\partial^2 E\pi}{\partial x^2} = \frac{h''[(a + r)V + x]}{(a + h + r)^2}$$
$$- \frac{2h'[(a + r)(Vh' - 1) - (h - xh')]}{(a + h + r)^3} < 0.$$

At $x = \hat{x}$, $(a + r)(Vh' - 1) - (h - xh') = 0$ so that $h'' < 0$ is necessary for $\partial^2 E\pi/\partial x^2 < 0$.

Define the random variable $\tau(n) = \min_{0 \leq i \leq n} \{\tau(\hat{x}_i)\}$. Then $\tau(n)$ is the random industry introduction time for the innovation. In equilibrium the expected value of $\tau(n)$ is given by $E\tau(n) = 1/nh(\hat{x})$. As a direct consequence of Theorem 1 we have

COROLLARY 1. An increase in the number of firms in the industry leads to an earlier expected industry introduction time for the innovation.

Proof. Note that

$$\frac{dE\tau(n)}{dn} = -h'\left(\frac{d\hat{x}}{dn}\right)\bigg/ nh^2.$$

Q.E.D.

COROLLARY 2. An increase in the number of firms in the industry decreases expected profits.

Proof. Define equilibrium profits as $\pi = \pi(a,\hat{x})$. Then

$$\frac{d\pi}{dn} = \frac{\partial\pi}{\partial a}\left[(n-1)h'\frac{d\hat{x}}{dn} + h\right] + \frac{\partial\pi}{\partial x}\frac{d\hat{x}}{dn}.$$

From (3), $\partial\pi/\partial a < 0$. Moreover, $\partial\pi/\partial x = 0$ by definition of $\hat{x}$. With $d\hat{x}/dn > 0$, this gives $d\pi/dn < 0$.

Q.E.D.

In spite of Theorem 1, Corollaries 1 and 2 yield results identical to Loury's.

As long as expected profits are positive, firms enter, increasing $\hat{x}$ and decreasing $E\pi$. This process stops when $E\pi = 0$. Using (1) and (2) this can be shown to hold if and only if $h'(x)[V - F] = 1$. Let the solution to this equation be denoted x^z. Define n^z as that value of n

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4. By definition, $\partial H/\partial a = d\hat{x}/da$. Hence

$$1 - \frac{\partial H}{\partial a}(n-1)h' = \frac{[(a+r)V + \hat{x}]h'' + (Vh' - 1)(n-1)h}{[(a+r)V + \hat{x}]h''}.$$

Now $h'' < 0$ at equilibrium so $1 - (\partial H/\partial a)(n-1)h' > 0$ if and only if

$$[(a+r)V + \hat{x}]h'' + (Vh' - 1)(n-1)h' < 0.$$

But note that

$$\frac{d\{\partial E\pi/\partial x\}}{dx} = \frac{(n-1)h'(Vh'-1) + [(a+r)V + x]h''}{(a+h+r)^2}.$$

Hence $d\{\partial E\pi/\partial x\}dx < 0$ is equivalent to $1 - (\partial H/\partial a)(n-1)h' > 0$. This is the sense in which the latter is interpreted as a "stability" condition. Heuristically, it requires that if a firm's competitors all increase their investments in R & D just enough to generate a unit increase in rivalry, then the remaining firm must respond with less than a full unit increase in its R & D effort.

such that $\hat{x}(n^z) = x^z$. Then (x^z, n^z) give the long-run (zero profits) investment in R & D and number of firms for the industry.[5]

III

We would like to compare the results obtained so far with a situation in which a monopolist controls the industry, but may wish to invest in a number of parallel R & D projects. As usual, the monopolist maximizes expected profits.

Assuming no economies of scale in operating R & D projects, we have

THEOREM 2. Let the number of firms be such that $E\pi \geq 0$ in the noncooperative equilibrium. Then a monopolist operating the same number of projects will make a total investment in R & D that is less than the aggregate noncooperative investment.

Proof. The monopolist sets x to maximize expected total profits. The first-order condition for this is

$$(5) \qquad \frac{\partial nE\pi}{\partial x} = \frac{n[r(Vh' - 1) - n(h - xh')]}{(nh + r)^2} = 0.$$

Let the solution to (5) be denoted $x^m(n)$. Now consider $\partial E\pi/\partial x$ evaluated at x^m. From (2),

$$\left.\frac{\partial E\pi}{\partial x}\right|_{x=x^m} = \frac{(a + r)[Vh'(x^m) - 1] - [h(x^m) - x^m h'(x^m)]}{[a + h(x^m) + r]^2}.$$

Substituting from (5),

$$\left.\frac{\partial E\pi}{\partial x}\right|_{x=x^m} = \frac{[(a + r)n - r][h(x^m) - x^m h'(x^m)]}{n[nh(x^m) + r]} > 0.$$

Since $E\pi$ is assumed to be concave in x, this implies that $x^m(n) < \hat{x}(n)$.

Q.E.D.

Of course the monopolist can set n as well as x. Maximizing $nE\pi$ with respect to n yields a final theorem.

THEOREM 3. A monopolist will set n^m less than n^z (the zero-profit number of firms in the noncooperative model). Moreover, $x^m(n^m) < \hat{x}(n^z) = x^z$.

5. We simply assume that $n^z \geq 1$. This reduces to an implicit constraint on V and F through h.

Proof. The first-order condition associated with n^m is

(6) $$\frac{\partial nE\pi}{\partial n} = \frac{(Vh - x)r}{(nh + r)^2} - F = 0.$$

Thus,

$$(Vh - x)/(n^mh + r) = F(n^mh + r)/r.$$

This implies that

(7) $$\frac{\partial nE\pi}{\partial n}\bigg|_{n=n^m} = \frac{(n^m)^2h}{r\,F} > 0$$

But

$$\frac{\partial nE\pi}{\partial n}\bigg|_{n=n^z}^{x=\hat{x}(n^z)} = 0.$$

Therefore,

$$\frac{\partial nE\pi}{\partial n}\bigg|_{n=n^z}^{x=x^m} \leq 0.$$

Hence $n^m < n^z$, since $\partial E\pi/\partial n < 0$. Moreover, $d\hat{x}/dn > 0$ implies that $\hat{x}(n^m) < \hat{x}(n^z) = x^z$. Together with Theorem 2 this yields $x^m(n^m) < \hat{x}(n^z)$.

Q.E.D.

By assuming that successful innovators act as perfectly discriminating monopolists, Loury interprets $nE\pi$ as the social value of innovation. In this case n^m and x^m are socially optimal values for n and x. Hence our model yields welfare conclusions identical to Loury's when h has an initial range of increasing returns.

IV

The model analyzed in this paper emphasizes the importance of variable costs in the R & D technology, while Loury's model focuses on the role of fixed costs. A natural extension of this entire line of research is to allow both fixed and variable costs to be set endogenously by the firms in the market. That is, the function relating investment in R & D to a random innovation introduction time could be defined over both F and x; $h = h(F,x)$. In this case ambiguous results are likely to arise regarding the relationship between rivalry and an individual firm's investment in R & D. However, if fixed costs are

more important than variable costs in the R & D technology (in some appropriate sense), then an increase in rivalry should lead to a decrease in the equilibrium level of firm investment in R & D. Similarly, if variable costs are more important than fixed, then an increase in rivalry should lead to an increase in the equilibrium level of firm investment in R & D.

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