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Exposita Notes

A short proof of the Bulow-Klemperer auctions vs. negotiations result*

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Summary. Bulow and Klemperer [1] have provided an upper bound on the value of bargaining power for a seller of an indivisible object. Specifically, negotiating optimally with N buyers yields lower revenue than an English auction with $N+1$ buyers. In this paper, a short and intuitive proof of this result is presented.

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Bulow and Klemperer [1] examine the value of bargaining power for the seller of an indivisible good. At best, the seller can use his bargaining power to negotiate optimally and attain the fully optimal revenue.[1] On the other hand, running an English auction with no reserve price demands no bargaining power on the part of the seller. With symmetric buyers, Bulow and Klemperer [1] show that the English auction with $N+1$ buyers yields strictly higher revenue than an optimal mechanism with N buyers. Hence, the value of bargaining power is bounded above by the value of an extra buyer. The aim of this paper is to provide a new, intuitive proof of this result.

Thus, consider a risk neutral seller with an indivisible object for sale, and normalize to zero the value the seller puts on consuming the good himself. The seller's objective is to maximize revenue. In the following, a mechanism that maximizes revenue among all mechanisms in which the object is sold with *certainty* will be

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[1] In Bulow and Klemperer's [1] terminology, optimal negotiation does not refer to a specific selling procedure, but rather to an optimal mechanism. In this paper, we will generally, with the exception of the English auction, abstract from the details of the selling procedure, by referring to it simply as a mechanism.

said to be *constrained optimal*. In a fully optimal mechanism, on the other hand, the object is not necessarily sold.[2]

Before the seller stages a mechanism, each buyer receives a private signal from some, commonly known, joint distribution. The value of the good to a given buyer can depend on his own signal, and on the signals of the other buyers. If a buyer loses the auction, he is indifferent between who, if any, wins. Finally, all buyers are *serious* in the sense of Bulow and Klemperer [1]. That is, regardless of signals, the valuation of any buyer is not below the seller's valuation. The buyers are divided into two disjoint and non-empty sets, the initial set of buyers and the set of newcomers, respectively. A member of the second set is a *newcomer*.

To prove Bulow and Klemperer's [1] result, we combine two simple observations, the first of which can be made at the present level of generality.

(i) The seller is at least as well off with a constrained optimal mechanism with the initial set of buyers and a newcomer as with an optimal mechanism with the initial set of buyers.

To see this, notice that an optimal mechanism with the initial set of buyers yields the same expected revenue as the following mechanism with the initial set of buyers and a newcomer. Simply stage an optimal mechanism with the initial set of buyers, and if the object is not sold in this mechanism, give the object to the newcomer.[3] However, since the good is sold with probability one in this new mechanism, expected revenue is at most as high as expected revenue in a constrained optimal mechanism.

The second observation we will make is less general. Thus, we start by imposing the assumptions of Bulow and Klemperer [1]. Buyers are symmetric and risk neutral, and signals are independent and drawn from a continuous distribution function. Furthermore, a buyer's valuation is bounded above, increasing in his own signal, depends symmetrically on the signals of the other buyers, and is non-decreasing in these. Hence, the model is the same as in Milgrom and Weber [7], but with *independent signals*. Finally, the *regularity assumption* is imposed. That is, the buyer with the highest signal also has the highest marginal revenue.[4] We refer to Bulow and Klemperer [1] for a detailed, and more formal, description of the assumptions.

(ii) Given the assumptions in Bulow and Klemperer [1], the English auction is constrained optimal.

To see this, notice that in this model the English auction is won by the buyer with the highest signal, as shown by Milgrom and Weber [7]. Furthermore, since signals are independent, a key result from Myerson's [8] seminal analysis of optimal

[2] To implement a constrained optimal mechanism, the seller generally needs the ability to *discriminate* (enforce asymmetric rules) and, in order to implement a fully optimal mechanism, the additional ability to *ration supply*. With symmetric buyers, the former ability is redundant.

[3] The newcomer, a serious buyer, is willing to pay a price of zero (the seller's valuation).

[4] Here, marginal revenue is the marginal increase in expected revenue from marginally increasing the probability that the buyer wins the object, given buyers have the vector of signals in question. See Myerson [8], Bulow and Roberts [2] or Bulow and Klemperer [1].

mechanisms can be utilized. *Specifically, a mechanism is constrained optimal if, and only if, the buyer with the highest marginal revenue wins.*[5] Hence, the English auction is constrained optimal.

On the other hand, the intermediate mechanism used in the proof of the first observation is *not* constrained optimal since the newcomer does not necessarily win when he has the highest signal. Consequently, the English auction yields *strictly* higher revenue. This concludes the proof.

Proposition 1 (Bulow and Klemperer) *An English auction with $N + 1$ buyers yields strictly higher expected revenue than an optimal mechanism with N buyers, given the assumptions in Bulow and Klemperer [1].*[6,7]

It must be pointed out that McAfee and McMillan [6] provide intuition that, though less developed, in part coincides with the intuition behind the proof presented here.[8] In particular, they observe that: "The bargainer's main source of bargaining power, the threat to refuse to sell the item [...] is analogous to another bidder in the competitive case. Yet, a real competitor is more effective than a fake one".

Finally, in a hybrid auction/bargaining model, Wang [9] argues that the ability to *renegotiate* the outcome of an auction may be preferable to more buyers. However, in this model the difference between the mechanisms that are compared lies in the ability to bargain after information is revealed, not in the ability to ration supply. Hence, observation *(i)* is not useful.

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[5] Strictly speaking, it is also necessary that a buyer with the lowest possible signal earns zero payoff from participating in the mechanism. This is trivially satisfied in the English auction with symmetric buyers, since such a buyer has zero probability of winning.

[6] With *asymmetric* buyers with *independent private values*, we can generalize the result: Given the ability to discriminate (and the information to do so), adding a *serious competitor* is *strictly* more valuable than the ability to ration supply, where a serious competitor is one who, with positive probability, has a higher valuation than the initial set of buyers. Then, since marginal revenue equals valuation "at the top" and is strictly lower elsewhere, the newcomer may be the participating buyer with the highest marginal revenue, yet he may lose in the intermediate mechanism used in the proof of observation *(i)*. Therefore, this mechanism is strictly inferior to a constrained optimal mechanism. Note, however, that the English auction is not constrained optimal with asymmetric buyers.

[7] When signals are *affiliated*, an optimal mechanism generally extracts all surplus, and the good is always sold (see Cremer and McLean [4,5] for details). Hence, the ability to ration supply is worthless. Nevertheless, Bulow and Klemperer [1] show that the English auction with $N + 1$ symmetric buyers is superior to any mechanism with N buyers where the winner, if any, is the buyer with the highest signal and where only the winner pays. Interestingly, Compte and Jehiel [3] show that with asymmetric buyers, a newcomer may cause welfare in an English auction to decrease.

[8] I would like to thank an anonymous referee for drawing my attention to this paper.

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