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LAPM: A Liquidity-Based Asset Pricing Model

BENGT HOLMSTRÖM and JEAN TIROLE*

ABSTRACT

The intertemporal CAPM predicts that an asset's price is equal to the expectation of the product of the asset's payoff and a representative consumer's intertemporal marginal rate of substitution. This paper develops an alternative approach to asset pricing based on corporations' desire to hoard liquidity. Our corporate finance approach suggests new determinants of asset prices such as the distribution of wealth within the corporate sector and between the corporate sector and the consumers. Also, leverage ratios, capital adequacy requirements, and the composition of savings affect the corporate demand for liquid assets and, thereby, interest rates.

STARTING WITH THE CAPITAL ASSET PRICING MODEL (CAPM, derived by Sharpe (1964), Lintner (1965), and Mossin (1966)), market finance has emphasized the role of consumers' time preference and risk aversion in determining asset prices. The intertemporal consumption-based asset pricing model (e.g., Rubinstein (1976), Lucas (1978), Breeden (1979), Harrison and Kreps (1979), Cox, Ingersoll, and Ross (1985), Hansen and Jagannathan (1991)) predicts that an asset's current price is equal to the expectation, conditioned on current information, of the product of the asset's payoff and a representative consumer's intertemporal marginal rate of substitution (IMRS). While fundamental, this dominant paradigm for pricing assets has some well-recognized shortcomings (see below), and there is clearly scope for alternative and complementary approaches. This paper begins developing one such approach based on aggregate liquidity considerations.¹

Our starting point is that the productive and financial spheres of the economy have autonomous demands for financial assets and that their valuations for these assets are often disconnected from the representative consumer's. Corporate demand for financial assets is driven by the desire to hoard liquidity in order to fulfill future cash needs. In contrast with the

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¹ Liquidity in this paper does not refer to the ease with which assets can be resold, but rather to the aggregate value of financial instruments used to transport wealth across time and to back up promises of future payments. While transaction costs (taxes, brokerage fees, etc.) have an important impact on the pricing of individual assets, their implications for aggregate liquidity have not yet been elucidated (for bid-ask spreads, see Amihud and Mendelson (1986)).

logic of traditional asset pricing models based on perfect markets, corporations are unable to raise funds on the capital market up to the level of their expected income, and hence the corporate sector will need a cushion against liquidity shocks (Holmström and Tirole (1996, 1998)). Financial assets that can serve as a cushion will command liquidity premia.²

There is substantial evidence that firms and banks hold liquid assets (e.g., Crane (1973), Harrington (1987), and, especially, Opler et al. (1999)). Companies protect themselves by holding securities and, particularly by securing credit lines and loan commitments from banks and other financial institutions. Lines of credit cover working capital needs and back up commercial paper sales. Commitments provide long-term insurance through revolving credits, which often include an option to convert the credit into a term loan at maturity, and through back-up facilities that protect firms against the risk of being unable to roll over their commercial paper. Companies pay a price for these insurance services through up-front commitment fees and costly requirements to maintain compensatory balances.

Turning to the supply side, the provision of liquidity is a key activity of the banking sector. Banks incur a nonnegligible credit risk, as the financial condition of companies may deteriorate by the time they utilize their credit facilities. Furthermore, the use of credit facilities varies substantially over time. Credit use tends to increase when money is tight, forcing banks to scramble for liquidity in order to meet demand. Banks themselves purchase insurance against unfavorable events. On the asset side, they hoard low-yielding securities such as Treasury notes and high-grade corporate securities. On the liability side, they issue long-term securities to avoid relying too much on short-term retail deposits. Within the banking sector, liquidity needs are managed through extensive interbank markets.

In the standard consumption-based models, asset prices are driven entirely by the consumers' intertemporal marginal rates of substitution (IMRS). Only real allocations matter; the net supply of financial assets is irrelevant. This results in crisp predictions, but some of them seem to match the evidence poorly. For instance, consumption-based models predict too small equity premia (Mehra and Prescott (1985)) and Treasury bill discounts.³ They

² This theme relates to Hicks' (1967) notion of "liquidity preference" for monetary instruments and other close substitutes. He defines "reserve assets" as assets that are held to facilitate adjustments to changes in economic conditions and thus not only for their yield. For a historical perspective on the developments following Keynes' (1973), and Hicks' (1967) contributions to liquidity preference, see Cramp (1992) and Panico (1992).

³ As Aiyagari and Gertler (1991) note, "reasonably parametrized versions [of the intertemporal asset pricing model] tend to predict too low a risk premium and too high a risk-free rate," and do not account for the seven percent secular average annual real return on stocks and the one percent secular average return on Treasury bills. A number of papers have used nonseparable preferences to address the equity premium puzzle (with mixed success; see Shiller (1989) and Ferson (1995) for reviews), while the other puzzle—the low risk-free rate—has been largely ignored as pointed out by Weil (1989) and Aiyagari and Gertler (1991).

also suggest high covariation among asset prices, because there is a single price driver. But as Shiller (1989, pp. 346–348) notes, “prices of other speculative assets, such as bonds, land, or housing, do not show movements that correspond very much at all to movements in stock prices.” The consumption-based theory seems to say little about various stylized facts concerning the yield curve, such as its predominantly upward slope. While it is often argued that the term premium results from the price risk of long-term bonds, this argument cannot be based on the Consumption CAPM, because price risk as such only entails reshuffling of wealth among investors and hence cannot deliver risk premia. Nor does the theory provide reasons why long-term bonds have so high a volatility (Shiller 1989, Chapter 12.) As a final, related observation, the consumption-based approach has not been of much use in the important development of Autoregressive Conditional Heteroskedasticity (ARCH) models.⁴ ARCH models allow the covariance matrix of innovations to be state contingent, in order to fit the observations made by Mandelbrot (1963), Fama (1965), and Black (1976) that variances and covariances of financial asset prices change through time and that volatility tends to be clustered across time and across assets. The empirical success of ARCH models has not been matched by the theory.⁵

We view these shortcomings as an invitation to explore alternative paradigms, fully recognizing that the achievements of the elegant, simple consumption-based theory are significant and hard to match. Our purpose here is not to provide a well-developed model that fits the evidence better, but simply to illustrate how a corporate finance-based approach to asset pricing could eventually help to bridge the gap between theory and evidence. The key feature that makes our approach potentially useful is the feedback effect from the supply of assets to real allocations. This makes asset prices sensitive to new factors, including the supply of liquidity by the corporate sector and the government. Let us briefly review some of the consequences.

First, concerning the equity premium puzzle, we find that Treasuries and other high-grade securities may offer better insurance than do stocks against shortfalls in corporate earnings and other liquidity needs. To highlight this point, we assume that consumers are risk neutral, so that stocks and bonds would trade at par in the standard model. Yet, in our model, bonds command a liquidity premium relative to most stocks.

Second, by assuming risk-neutral consumers, we eliminate consumer IMRSs as drivers of asset-price movements. Instead, prices respond to changes in corporate IMRSs, which in turn are influenced by corporate demand for li-

⁴ See, for example, Bollerslev, Chou, and Kroner (1992), Engle, Ng, and Rothschild (1990) and Ghysels, Harvey, and Renault (1996). ARCH models, though, have been used on an ad hoc basis (for example by allowing the betas in a CAPM model to change over time) in order to improve the fit of the intertemporal asset pricing model.

⁵ A recent theoretical paper, which develops a model with serially correlated volatility, is Spiegel and Subrahmanyam (2000).

quidity. The theoretical implications are different. For instance, unlike the consumption CAPM, our model implies that an increase in the supply of liquidity drives bond prices down at the same time as stock prices go up.⁶ For empirical work, we believe that the corporate sector may offer a better measure of changes in the marginal purchaser's IMRS than do corresponding attempts to study partial participation among consumers (for a recent effort, see Vissing-Jørgensen (1997)).⁷

Third, in our model, the yield curve is determined by the value of bonds of different maturities as liquidity buffers, the availability of substitute instruments for this purpose, and the anticipated corporate and institutional liquidity needs. This generates richer patterns for the yield curve than in a consumption-based theory.

Fourth, in our model, price volatility is state contingent and exhibits serial correlation. Under some conditions, the price volatility of fixed-income securities covaries negatively with the price level (as Black (1976) notes, volatilities, which covary across assets, go up when stock prices go down). Intuitively, fixed-income securities embody an option-like liquidity service. When there is a high probability of a liquidity shortage, the option is "in the money" and its price will be sensitive to news about the future. When the probability of a liquidity shortage is sufficiently low, the price of the option goes to zero and will not respond much to news.

The paper is organized as follows. Section I illustrates the determination of liquidity premia in a simple example. Section II sets up a general model of corporate liquidity demand. This model shows how departures from the Arrow–Debreu paradigm generate liquidity premia. Sections III and IV demonstrate that the liquidity approach delivers interesting insights for the volatility of asset prices and for the yield curve.

I. Liquidity Premium: An Example

There are three periods, $t = 0, 1, 2$, one good, and a continuum (of mass 1) of identical entrepreneurs, each with one project. Entrepreneurs are risk

⁶ It is interesting, if premature, to take note of the striking drop in the yield of the 30-year Treasury bond in response to the government's plan to reduce their supply. There has not been a corresponding decline in the yields of long-term corporate bonds. This evidence is consistent with the view that government bonds provide important liquidity in critical states of nature, unlike corporate bonds. Of course, other possible explanations exist.

⁷ Cochrane (1991, 1996) presents a production-based asset pricing model, which demonstrates that the empirical implications stemming from the production side can be richer and fit the data better than the predictions coming out of standard consumption-based models. These models are quite different from ours in that they are not based on liquidity shortages, but instead assume complete markets. On the other hand, Cochrane works out in much more detail the empirical implications of studying the production side; in particular, the relationship between real investment returns and asset returns. It would be useful to approach some of the same empirical issues using our liquidity framework, but that is beyond the scope of the present paper.

neutral and do not discount the future. They have no endowment at date 0 and must turn to investors in order to defray the fixed date-0 set-up cost, I , of their project. At date 1, the project generates a random verifiable income x . The realization of x is the same for all entrepreneurs and so there is aggregate uncertainty. The distribution $G(x)$ of x is continuous on $[0, \infty)$, with density $g(x)$ and a mean greater than I .

At date 1, the firm has the opportunity to invest an additional amount $y \geq 0$, which at date 2 generates a payoff $by - y^2/2$; $b > 1$. Note that the firm is risk averse with respect to the reinvestment level y . The first-best reinvestment level equals $b - 1 > 0$.

A key feature of our model is that a portion of the firm's date-2 investment returns cannot be pledged to outside investors; there is a nonpledgeable part that accrues privately to the entrepreneur. In this section we assume for simplicity that none of the date-2 returns are pledgeable, so no financial claims on date-2 corporate assets can be written; in the general model a fraction of the date-2 returns can be pledged.⁸

We assume throughout the paper that the firm's date-1 return x is fully pledgeable (because having a portion of it nonpledgeable would not matter for the liquidity analysis) and that contracts between the entrepreneur and the investors can be made contingent on date-1 returns (as well as the pledgeable part of the date-2 returns in the general model to come). In other words, contingent claims markets are complete on the pledgeable portion of firm returns. The only market imperfection stems from the nonpledgeable part of the date-2 income.⁹

There is one noncorporate asset, land.¹⁰ Returns on land are fully pledgeable. Units of land are normalized so that there are \bar{L} units, each yielding at date 1 one unit of the good in every state (this unit of good can be thought of as the sum of the date-1 value of the crop harvested on the land plus the date-1 resale price.) The date-0 price of a unit of land is q .

Consumers (investors) are risk neutral and do not discount the future. They value the consumption stream (c_0, c_1, c_2) at $c_0 + c_1 + c_2$. Because consumers will buy any asset with a strictly positive expected rate of return, the land price must satisfy $q \geq 1$. As in our 1998 paper, we assume that

⁸ In our 1998 paper we offer a microeconomic rationalization for the nonpledgeable part: It provides a wealth-constrained entrepreneur the incentive to work diligently. A simpler interpretation is that the entrepreneur enjoys operating the firm, but cannot pay for it, because he is wealth constrained.

⁹ This contrasts with other liquidity-constrained models that feature imperfections at both dates (for instance, Krishnamurthy (1998), and Shleifer and Vishny (1992)).

¹⁰ The only feature of land that is going to be relevant for our analysis is that its payoff is exogenous. An alternative interpretation of this noncorporate financial asset is a Treasury bond: See our 1998 paper. One may, whenever we refer to Treasury bonds, think of them as backed up by government-owned land. Alternatively, and more realistically, the government bonds may be backed by taxpayer income. Taxation at date 1 (to meet the obligations on short-term bonds) and at date 2 (for the long-term bonds) need not introduce general equilibrium effects in our model since consumers' preferences are linear.

will hold as an equality and the entrepreneur will receive the full social surplus associated with production. An optimal contract can be determined by maximizing the entrepreneur's return:

$$E_0 \left[by(x) - \frac{(y(x))^2}{2} + t(x) \right], \quad (3)$$

subject to constraints (1) and (2). Letting μ denote the shadow price of constraint (2), this optimization amounts to

$$\max_{\{y(\cdot), L\}} \left\{ E_0 \left[by(x) - \frac{(y(x))^2}{2} + t(x) + \mu[x - I - y(x) - t(x) - (q - 1)L] \right] \right\} \quad (4)$$

subject to the liquidity constraint (1).

For fixed L , the solution of the unconstrained program is

$$y^* = b - \mu, \quad (5)$$

and the solution to the liquidity-constrained program is therefore

$$y(x) = \min(y^*, x + L). \quad (6)$$

The firm is liquidity constrained in low-income states $x < y^* - L$.

The multiplier for the budget constraint must satisfy $\mu \geq 1$; or else the solution to the unconstrained program would not pay investors anything at date 1, violating (2). If $\mu = 1$, the firm generates enough income at date 1 to pay back investors and still leave some surplus ($t(x) > 0$ for some x values). In this case (5) and (6) show that the first-best level of reinvestment will be chosen whenever liquidity so permits. If in addition $q = 1$, it costs nothing to carry liquidity and (1) can be satisfied for all x by choosing $L = b - 1$. This yields first-best for all x . We are not interested in a first-best outcome and will therefore assume that the budget constraint is binding, that is, (2) is violated when $L = b - 1$ and $y(x) \equiv b - 1$. It follows that $\mu > 1$ and, from the objective function of the unconstrained program, that $t(x) = 0$ for all x .

Let us turn to the date-0 choice of liquidity. From our previous characterization, L is chosen to maximize

$$\begin{aligned} & \int_0^{y^*-L} \left[b(x+L) - \frac{(x+L)^2}{2} - \mu(I+qL) \right] g(x) dx \\ & + \int_{y^*-L}^{\infty} \left[by^* - \frac{(y^*)^2}{2} - \mu[I+y^*+(q-1)L-x] \right] g(x) dx. \end{aligned} \quad (7)$$

Assuming $L > 0$ the first-order condition is

$$q - 1 = \int_0^{y^*-L} \left[\frac{b - (x+L)}{\mu} - 1 \right] g(x) dx. \quad (8)$$

Define the *marginal value of the liquidity service*:

$$m(x) \equiv \begin{cases} \frac{b - (x + L)}{\mu} - 1 & \text{for } x \leq y^* - L \\ 0 & \text{for } x \geq y^* - L. \end{cases} \quad (9)$$

Then

$$q - 1 = E_0[m(x)]. \quad (10)$$

The liquidity premium is equal to the expected marginal value of the liquidity service. In liquidity-shortage states ($x < y^* - L$), an extra unit of land allows the firm to increase its reinvestment by one and the private benefit by $b - (x + L)$. This marginal private benefit, expressed in monetary terms, is equal to $[b - (x + L)]/\mu$. The increase in reinvestment has monetary cost one. This yields the expression for the liquidity premium (10).

An equilibrium consists of a pair of prices $\{q, \mu\}$ and an optimal plan $\{L, y(\cdot)\}$ such that $t(x) = 0$ for all x , (2) holds with equality, the reinvestment policy $y(\cdot)$ satisfies (6), and the asset pricing equation (8) holds with $L = \bar{L}$ if $q > 1$, and $L \leq \bar{L}$ if $q = 1$. When $q = 1$, land commands no liquidity premium. In that case L can, without extra cost, be chosen equal to $b - 1$ to avoid liquidity constraints. It is easy to show that (i) there exists L^* such that $q > 1$ if and only if $\bar{L} < L^*$ and, (ii) the value of the marginal liquidity service $m(\cdot)$ and the price of liquid claims q are monotonically decreasing in \bar{L} . These results are illustrated in Figure 2, where $m_{\bar{L}}(\cdot)$ denotes the marginal liquidity service for land supply \bar{L} . When $\bar{L} = \bar{L}^3 > L^*$, the economy is replete with liquidity, there is no liquidity premium, and land price is low. The cases $\bar{L} = \bar{L}^1$ or \bar{L}^2 depict the interesting case of scarce liquidity.

The example shows that the value of the liquidity service can be viewed as a put option. The value of the put option is higher when there is less land, that is, less liquidity. This option feature is important for volatility. Note that the value of liquidity $m(x)$ is linear in the example. Linearity follows from the functional form of the date-2 return. In our 1998 paper, we use a specification in which the date-2 return can take on only two values and a fraction of it can be pledged to outsiders. Under some mild regularity conditions, $m(x)$ will be convex for this specification.¹²

¹² The assumptions are: The reinvestment y produces date-2 income $f(y)$ with probability p and 0 with probability $1 - p$. The entrepreneur can work ($p = p_H$) or shirk ($p = p_L = p_H - \Delta p$). Shirking generates a private benefit $Bf(y)$. Letting $\rho_0 \equiv p_H[1 - (B/\Delta p)]$; $m''(x) \geq 0$ if $3\rho_0(f'')^2 + (1 - \rho_0 f')f''' \geq 0$.

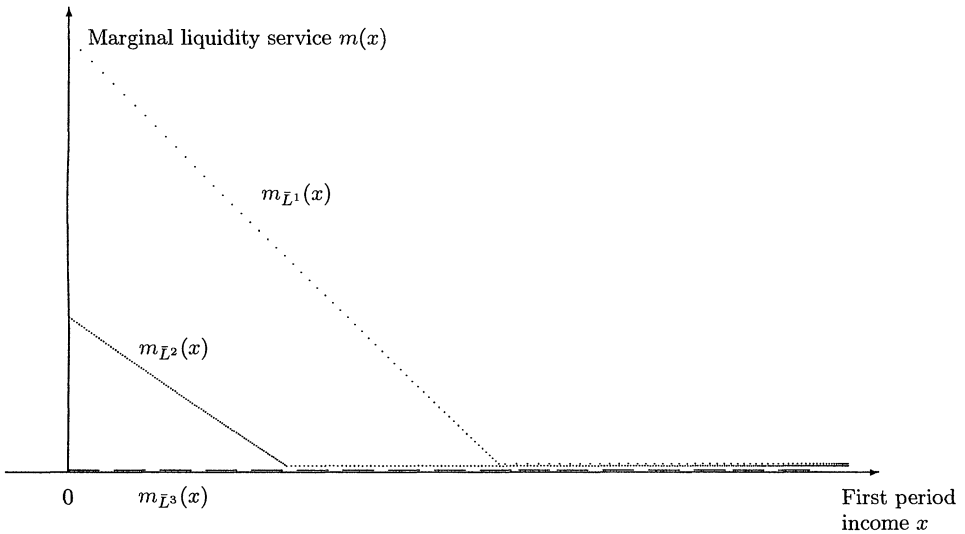


Figure 2. $\bar{L}^3 > L^* > \bar{L}^2 > \bar{L}^1$.

II. LAPM

Let us now develop a more general framework. There are three periods, $t = 0, 1, 2$. At date 1, a state of nature ω is revealed to all economic agents. There may be a further resolution of uncertainty at date 2, but in our risk-neutral framework, only date-1 expectations matter and so we need not specify any date-2 random events. The state of nature ω includes the incomes of the corporations at date 1 (as in the example above), their date-1 reinvestment needs and opportunities (as in our 1998 paper), and possibly other news.

A. Agents

As in the example, and in order to highlight the departure from the canonical asset pricing model, we assume that all agents are risk neutral and have an exogenously given discount rate, normalized at zero. That is, agents value consumption stream (c_0, c_1, c_2) at $c_0 + c_1 + c_2$. One could assume more generally that investors have endogenously determined and possibly stochastic discount factors. Similarly, the implicit assumption that consumers face no liquidity needs could be relaxed.

B. Noncorporate Claims

At date 0, there are K noncorporate assets, $k = 1, \dots, K$ such as land, real estate, or Treasury securities. The state-contingent return on asset

k at date 1, that is, the date-1 dividend plus the date-1 price, is equal to $\theta_k = \theta_k(\omega) \geq 0$. The mean return on each asset is normalized to be one: $E_0[\theta_k(\omega)] = 1$, where $E_t[\cdot]$ denotes the expectation of a variable conditional on the information available at date t . Let \bar{L}_k denote the supply of asset k . At date 0, asset k trades at price q_k per unit, where $q_k \geq 1$ from the nature of consumer preferences. The *liquidity premium* on asset k is equal to $q_k - 1$.

Note that claims $k = 1, \dots, K$ do not include claims on the corporate sector (shares, bonds, deposits, CDs, etc.). We will later provide valuation formulas for the latter.

C. Corporate Sector

Our model treats the productive and financial sectors as a single, aggregated entity, called the “corporate sector.” The corporate sector invests at dates 0 and 1 and receives proceeds at dates 1 and 2. Let I denote the corporate sector’s date-0 gross investment (or vector of gross investments) in productive (illiquid) assets. Its date-0 net investment, $N(I)$, is equal to the difference between the gross investment and the productive sector’s capital contribution at date 0 (in the example above, $N(I) = I$ since the entrepreneurs had no initial wealth).

We treat the corporate sector as a single entity, because it is analytically convenient: Prices and quantities can be determined by maximizing the corporate sector’s objective function, which is a weighted average of the objective functions of individual firms. The Appendix rationalizes this approach. The underlying assumption is that the institutional structure in the corporate sector is rich enough to admit efficient use of liquidity. Banks and securities markets exist to coordinate and redistribute liquidity in a manner that guarantees that each unit of scarce liquidity gets allocated to its best use across firms. In particular, no firm holds unused liquid claims in states of aggregate liquidity shortage.

D. Assumptions

ASSUMPTION 1 (Spanning): *The complete date-1 space of liquidity, generated from state-contingent Arrow–Debreu claims (paying one unit of liquidity in exactly one state ω), either is spanned by the traded assets $\{L_k\}$ or can be spanned by reshuffling liquidity on a state-contingent basis among firms within the corporate sector.*

Assumption 1 allows us, when convenient, to think of firms as buying liquidity in state-contingent form at state-dependent prices $m(\omega)$ that are the same for all firms. Assumption 1 could be weakened. We invoke it mainly to minimize the extent to which our model differs from the Arrow–Debreu model. The only significant difference now is that in our model, firms cannot pledge all the returns to investors (see below). On the pledgeable portion of firm income, however, markets are complete.

The net investment $N(I)$ is only part of the investors' date-0 contribution to the corporate sector. The corporate sector also purchases at date 0 non-corporate assets $\{L_k\}_{k=1,\dots,K}$. The investors' date-0 outlay is thus

$$N(I) + \sum_k q_k L_k. \quad (11)$$

Note that in equilibrium all claims commanding a liquidity premium ($q_k > 1$) must be held by the corporate sector ($L_k = \bar{L}_k$). This is, of course, an artifact of the assumption that consumers have no liquidity demand.

At date 1, the corporate sector selects a policy or decision $d = d(\omega)$ in a feasible set $D(I, \omega, L(\omega))$, where $L(\omega)$ is the net liquidity available to the corporate sector in state of nature ω . The decision vector d includes all real decisions within the corporate sector such as reinvestments and production decisions for each firm. Implicitly it also includes reallocations of income and liquidity to support these real decisions. If the corporate sector does not pay insiders (entrepreneurs) anything at date 1 (see below), then

$$L(\omega) = \sum_k \theta_k(\omega) L_k. \quad (12)$$

Note that at date 0 the investors cannot commit to bringing in new funds at date 1 beyond the amount that is backed up by the liquid assets $L(\omega)$.¹³

ASSUMPTION 2 (Opportunity-enhancing Liquidity): For all ω, L, L' : if $L < L'$, then $D(I, \omega, L) \subseteq D(I, \omega, L')$.

In general, an increase in liquid reserves *strictly* enlarges the set of feasible corporate policies when financial markets are imperfect (in the example, $I, \omega = x$, and $D(I, \omega, L) = \{y | y \leq x + L\}$).

For a given state of nature ω at date 1, let $R(I, \omega, d)$ denote the *total expected intertemporal* (i.e., date 1 plus date 2) *payoff* from illiquid corporate assets (in the example, $R = x + [by - (y^2/2)] - y$). R includes pledgeable and nonpledgeable returns on illiquid assets, but excludes the return $\sum_k \theta_k(\omega) L_k$ on noncorporate securities. Similarly, let $r(I, \omega, d)$ denote the *net pledgeable income* from illiquid assets, that is, the total expected intertemporal income that can be promised to date-0 investors net of date-1 reinvestments (in the example $r(I, \omega, d) = x - y$). Accounting for the noncorporate assets purchased by the corporate sector at date 0, the corporate sector can thus pay out

$$r(I, \omega, d(\omega)) + \sum_k \theta_k(\omega) L_k \quad (13)$$

¹³ See our 1998 paper for a discussion of this assumption, which is closely related to that of no (uncollateralized) short sales.

to investors. Let

$$B(I, \omega, d(\omega)) \equiv R(I, \omega, d(\omega)) - r(I, \omega, d(\omega)) \quad (14)$$

denote the nonpledgeable portion of income. The assumption that $B > 0$ is critical for generating a corporate demand for liquidity in our model.

Let $t(\omega)$ denote the part of *pledgeable* income that is paid out (at date 1) to corporate insiders (entrepreneurs) in state ω . The net liquidity available to support reinvestment in state ω is then $\sum_k \theta_k(\omega) L_k - t(\omega)$.

The corporate sector solves:

$$\max_{\{I, L, d(\cdot), t(\cdot)\}} \{E_0[B(I, \omega, d(\omega)) + t(\omega)] - [I - N(I)]\} \quad (15)$$

subject to the investors' break-even condition:

$$E_0[r(I, \omega, d(\omega)) - t(\omega)] + \sum_k L_k \geq N(I) + \sum_k q_k L_k, \quad (16)$$

and to date-1 decisions being feasible:

$$d(\omega) \in D\left(I, \omega, \sum_k \theta_k(\omega) L_k - t(\omega)\right). \quad (17)$$

Because the investors' break-even constraint is binding,¹⁴ this program can be rewritten as the maximization of the corporate sector's NPV:

$$\max_{\{I, L, d(\cdot), t(\cdot)\}} \{E_0[R(I, \omega, d(\omega))] - I - \sum_k (q_k - 1) L_k\}, \quad (18)$$

subject to (16) and (17).

Let $\mu \geq 0$ denote the shadow price of the break-even constraint in the former program, and let

$$m(\omega) \equiv \frac{d}{dL} \left[\max_{d \in D(I, \omega, L)} \left\{ \frac{B(I, \omega, d) + \mu r(I, \omega, d)}{\mu} \right\} \right] \Big|_{L=\bar{L}(\omega)} \quad (19)$$

denote the *marginal liquidity service* (expressed in terms of pledgeable income) in state ω assuming that the available liquidity is $\bar{L}(\omega) \equiv \sum_k \theta_k(\omega) \bar{L}_k$. Assumption 2 implies that $m(\omega) \geq 0$ for all ω .

Assume that there exists at least one state of nature in which there is excess liquidity, that is, in which the decision $d(\omega)$ is in the interior of the feasible decision set D . This is a mild assumption and is satisfied in all our examples. It implies (see the Appendix for more detail) that pledgeable in-

¹⁴ If it were not binding, the corporate sector could increase the investors' net contribution $N(I)$ without violating (16) or affecting (17).

come is never redistributed to the corporate sector in states of liquidity shortage ($t(\omega) = 0$ if $m(\omega) > 0$), and so the available liquidity in equation (19) is appropriately defined.¹⁵

Optimization with respect to L_k yields equilibrium prices for the noncorporate claims¹⁶

$$q_k - 1 = E_0[\theta_k(\omega)m(\omega)], \quad (20)$$

or, equivalently

$$q_k = E_0[\theta_k(\omega)[1 + m(\omega)]]. \quad (21)$$

Like risk premia, liquidity premia are determined by a covariance formula, but this time involving the intertemporal marginal rate of substitution $1 + m(\omega)$ of the corporate sector. An asset's liquidity premium is high when it delivers income in states in which liquidity has a high value for the corporate sector.

For completeness, we can finally introduce *claims issued by the corporate sector* (shares, bonds, etc.). Let \bar{L}_j be the date-0 supply of claim j paying $\bar{\theta}_j(\omega)$ at date 1 in state of nature ω . The set of corporate claims, $j = 1, \dots, J$, must satisfy

$$\sum_j \bar{\theta}_j(\omega) \bar{L}_j = r(I, \omega, d(\omega)) + \sum_k \theta_k(\omega) L_k - t(\omega). \quad (22)$$

The date-0 prices of such claims, $\{q_j\}_{j=1, \dots, J}$, will be given by

$$q_j - 1 = E_0[\bar{\theta}_j(\omega)m(\omega)]. \quad (23)$$

Firms will issue claims to match their production plans. A firm maximizes its market value by issuing securities that pay investors back in states in which liquidity is costly, the firm's income is high, and its investment returns are low. Note that firms must issue a sufficiently rich set of securities (or have intermediaries synthetically create them) for Assumption 1 to hold (in our 1998 paper we show that if a firm does not issue a sufficiently rich set of securities, other firms can free ride on its liquidity reserves by holding its shares, leading to an inefficient use of aggregate liquidity).

¹⁵ By the same reasoning, we will be able to ignore state-contingent liquidity withdrawals $t(\cdot)$ in the other programs in the paper.

¹⁶ Note that we assume that the corporate sector as a whole takes the prices of noncorporate assets as given. Price taking presumes that there is competition for assets within the corporate sector.

D.1. Single State of Liquidity Shortage

Suppose liquidity is scarce in a single state, ω_H , which has probability f_H . According to (20), the liquidity premium on asset k is then proportional to the asset's payoff conditional on the occurrence of the bad state:

$$q_k - 1 = f_H \theta_k(\omega_H) m_H. \quad (24)$$

This linear relationship yields, for any other asset ℓ ,

$$\frac{q_k - 1}{q_\ell - 1} = \frac{\theta_k(\omega_H)}{\theta_\ell(\omega_H)} \quad (25)$$

which implies a one-factor model of liquidity premia. We can take a bond price as the single factor. That is, if q_b is the date-0 price of a bond delivering for sure one unit of good at date 1, the liquidity premium on asset k is proportional to the liquidity premium on the bond:¹⁷

$$q_k - 1 = \theta_k(\omega_H)(q_b - 1). \quad (26)$$

With more than one state of scarce liquidity, (20) results in a multifactor model where the factors can be chosen as the liquidity premia of any subset of assets that spans the states in which liquidity shortages occur.

D.2. The Arrow–Debreu Economy

To underline the importance of the wedge $B > 0$ between net pledgeable income r and total income R , let us briefly consider the alternative case $B = 0$.

ASSUMPTION 3 (Fully-pledgeable Income): *For all $I, \omega, d, R(I, \omega, d) = r(I, \omega, d)$.*

Assumption 3 eliminates agency costs arising from private (nonpledgeable) benefits. As a consequence, liquidity does not directly affect pledgeable income.

OBSERVATION (Efficient Contracting): *For all ω and all L ,*

$$\max_{d \in D(I, \omega, L)} r(I, \omega, d) = \max_{d \in D(I, \omega, 0)} r(I, \omega, d). \quad (27)$$

To see this, suppose that the corporate sector hoards no liquidity, and so $L = 0$. Suppose that in state of nature ω , there exists L such that

$$r(I, \omega, d^*(I, \omega, L)) > r(I, \omega, d^*(I, \omega, 0)), \quad (28)$$

¹⁷ This assumes that the asset's payoff θ_k does not vary conditional on ω_H . If it varies, then $\theta_k(\omega_H)$ in (26) should be replaced by $E(\theta_k | \omega_H)$.

where $d^*(I, \omega, L)$ denotes the decision that maximizes pledgeable income in state ω given liquidity L . The corporate sector could at date 1 borrow L and pledge $r(I, \omega, d^*(I, \omega, L)) - L + L > r(I, \omega, d^*(I, \omega, 0))$, which contradicts the optimality of $d^*(I, \omega, 0)$.

Given Assumption 2, this observation implies that $m(\omega)$ is equal to zero for all ω , since

$$\begin{aligned} \max_{d \in D(I, \omega, L(\omega))} \left\{ \frac{B(I, \omega, d) + \mu r(I, \omega, d)}{\mu} \right\} &= \max_{d \in D(I, \omega, L(\omega))} \{r(I, \omega, d)\} \\ &= \max_{d \in D(I, \omega, 0)} \{r(I, \omega, d)\}. \end{aligned} \quad (29)$$

Thus, all liquidity premia are zero in an Arrow–Debreu economy.

III. Information Filtering and Volatility

As we noted in the beginning of the paper, many recent advances in empirical finance were motivated by the observation that conditional variances and covariances change over time. It is well known, for instance, that volatility is clustered, that asset volatilities (stock volatilities, bond volatilities across maturities) move together, and that stock volatility increases with bad news.¹⁸ This section does not attempt to provide a general theory of the impact of liquidity premia on volatility. Its only goal is to suggest that a liquidity-based asset pricing model has the potential to deliver interesting insights into state-contingent volatilities.

A. Example

Let us first return to the example of Section I. In this example with non-verifiable second-period income, the liquidity benefit of holding land is a put option, since $m(x)$ decreases linearly with first-period income x until it hits zero. We also observed that with partially verifiable second-period income and under some regularity conditions, $m(x)$ decreases and is convex until it hits zero.

Suppose now that news arrives intermittently between dates 0 and 1 containing information about the realization of x at date 1. Specifically, suppose that there are N news dates between 0 and 1 (the first distinct from date 0

¹⁸ Black (1976) attributes the last fact to the “leverage effect” (equity, which is a residual, moves more when the debt-equity ratio increases). On the other hand, this leverage effect does not seem to account for clustering of volatility and comovements between stock and bond volatilities.

and the N th equal to date 1). Assume further that the realization of x is given by either an additive or a multiplicative process

$$\begin{aligned} x &= x_0 + \sum_{m=1}^N \eta_m \\ &\equiv x_n + \sum_{m=n+1}^N \eta_m \quad \text{for all } n, \end{aligned} \quad (30a)$$

or

$$\begin{aligned} x &= x_0 [\times_{m=1}^N \eta_m] \\ &\equiv x_n [\times_{m=n+1}^N \eta_m] \quad \text{for all } n, \end{aligned} \quad (30b)$$

where the increments η_m are independently and identically distributed with mean zero. At subdate n , η_n is revealed and hence x_n contains all the relevant information about x .

The early accrual of information about the state ω will have no impact on the optimal decision rule $d(\cdot)$ and hence no retrading of financial contracts occurs between dates 0 and 1. Yet, we can price land by arbitrage at each subdate n . Contingent on the available information, summarized by x_n , at subdate n ,

$$q_n(x_n) - 1 = E_n[m(x)|x_n], \quad (31)$$

where $E_n[\cdot|\cdot]$ denotes the conditional expectation given information at subdate n . (Formula (31) is derived formally for the general framework in Section III.B.)

Both for the additive (30a) and the multiplicative (30b) processes, it can be shown that,¹⁹

$$\frac{d}{dx_n} [E_n[[q_{n+1}(x_{n+1}) - q_n(x_n)]^2 | x_n]] \leq 0. \quad (32)$$

In other words, the *volatility* of the price of land is state contingent.²⁰ Volatility is low when date-1 liquidity is expected to be plentiful and it grows higher the greater the fear of a liquidity shortage. In the example, liquidity is high when date-1 firm income is high, so volatility and expectations about the future of the economy are negatively related.

¹⁹ The result is true whenever $m' < 0$ and $m'' \geq 0$. The proof for the multiplicative process follows from that for the additive process by taking logs in (30b). Indeed if $m(\cdot)$ is decreasing and convex, $m(e^{\hat{x}})$ is also decreasing and convex in \hat{x} , where $\hat{x} \equiv \log x$.

²⁰ We should stress that the result in (32) refers to price volatility, not return volatility. The volatility of the return on land claims may well be increasing in x_n , since q_n is nondecreasing. Indeed, this will be the case when there is a single state of liquidity shortage; see (45).

The logic behind the positive relationship between price volatility and economic downturns follows from our earlier interpretation of $m(x)$ as an option (Figure 2). Volatility is higher when an option is in the money than when it is out of the money. If the information process were a continuous time geometric Brownian motion, one could use the Black–Scholes formula to derive an explicit expression for the volatility of bond prices in (32).

B. Contingent Volatility Formulae

Let us investigate more generally the impact of news about the state of nature in the LAPM framework of Section II. As in the example, we assume that there are subdates $n = 1, \dots, N$ between dates 0 and 1, at which informative signals accrue about the date-1 state of nature ω . Thus, the market's information about the state of nature at date 1 gets refined over time. Let σ_n denote the market's information at subdate n with $\sigma_N = \omega$. Let $E[\cdot | \sigma_n]$ denote the expectation of a variable conditional on the information available at time n . The corporate sector purchases quantity L_k of liquid asset k at date 0, and can afterwards reconfigure its portfolio so that it holds (information contingent) quantity $L_k(\sigma_n)$ at subdate n . Asset k 's equilibrium price given information σ_n is denoted $q_k(\sigma_n)$.

Consider the problem of maximizing the corporate sector's expected payoff subject to the investors' date-0 break-even condition and date-1 decisions being feasible:

$$\max_{\{I, L, L(\cdot), d(\cdot)\}} \{E_0[B(I, \omega, d(\omega))] + N(I) - I\}, \quad (33)$$

subject to

$$\begin{aligned} E_0[r(I, \omega, d(\omega))] &\geq N(I) + \sum_k q_k L_k \\ &+ E_0 \left[\sum_k \sum_{n=1}^N q_k(\sigma_n) [L_k(\sigma_n) - L_k(\sigma_{n-1})] \right] \\ &- E_0 \left[\sum_k \theta_k(\omega) L_k(\omega) \right], \end{aligned} \quad (34)$$

and

$$d(\omega) \in D \left(I, \omega, \sum_k \theta_k(\omega) L_k(\omega) \right). \quad (35)$$

A few comments are in order. First, σ is measurable with respect to ω , and so the set of feasible decisions is indeed a well-defined function of the state of nature. Second, the date-0 contract with investors specifies some portfolio

adjustment at each date. We ignore the possibility that contemplated portfolio adjustments may require a net contribution by investors at subdate n ($\sum_k q_k(\sigma_n)[L_k(\sigma_n) - L_k(\sigma_{n-1})] > 0$). While such a contribution could occur if the portfolio adjustment raised the investors' wealth conditional on σ_n , it would not occur if the adjustment reduced it, since the investors would be unwilling ex post to bring in new funds, and they cannot ex ante commit to do so. However, if in equilibrium $q_k > 1$, then $L_k(\sigma_n) = \bar{L}_k$ for all σ_n is an optimal policy, and so investors do not have to contribute at intermediate dates. The fictitious subdate- n reshuffling of liquid assets between the corporate sector and the rest of the economy is, as in Lucas (1978), only meant to price financial assets at an intermediate date.

As before, we let μ be the multiplier of the break-even constraint in (34) and define the marginal liquidity service $m(\omega)$ as in (19). Taking first-order conditions of the Lagrangian we find that for each liquid asset $k \in \{1, \dots, K\}$ and for each subdate $n \in \{1, \dots, N - 1\}$

$$q_k = 1 + E_0[\theta_k(\omega)m(\omega)],$$

$$q_k = E_0[q_k(\sigma_n)]$$

and

$$q_k(\sigma_n) = E_n[\theta_k(\omega)[1 + m(\omega)|\sigma_n]. \quad (36)$$

Asset prices (and liquidity premia) form a martingale because there is no liquidity service in subperiods where news arrives.²¹ Only at the last subdate (date 1) will the liquidity premium disappear and hence the martingale property fail. The martingale condition reflects the fact that firms are indifferent regarding the timing of the purchase of liquidity, as long as the purchase is made before the final date, that is, it is a consequence of arbitrage within the investors' budget constraint.

Define a "generalized fixed-income security" as one with expected return unchanged as news accrues:

$$E_n[\theta_k(\omega)|\sigma_n] = 1 \quad \text{for all } n \text{ and } \sigma. \quad (37)$$

For simplicity, we focus on these securities in the rest of this section.²² We can write (36) as

$$q_k(\sigma_n) - 1 = E_n[\theta_k(\omega)m(\omega)|\sigma_n]. \quad (38)$$

²¹ In a general T-period model with liquidity services and news accruing each period, prices would fall on average. Separating dates of news accrual and liquidity services has the advantage of a clean analysis of news-induced volatility of liquidity premia.

²² An indexed Treasury bond is the prime example we have in mind.

Condition (37) rules out volatility stemming from news about the assets' dividends. Such volatility must be added (with a correction depending on the covariance with the innovations about liquidity needs) to the price formulae (38) when expected payoffs change over time.

Note that if $\theta_k(\omega)$ is nonpositive for all ω for which $m(\omega) > 0$ (as is the case for a claim on the corporate sector as a whole), the "liquidity premium" in (38) will in fact be a discount on the fundamental value of the asset ($q_k(\sigma_n) < 1$). This implies that, in response to news about the demand for liquidity, aggregate bond and equity prices will move in opposite directions. By contrast, a change in the supply of bonds has the traditional implication that bond and equity prices comove positively.

C. Contingent Volatility and Clustering in the Case of a Single State of Liquidity Shortage

Assuming that there is a single state (state ω_H) in which there is a liquidity shortage, let $f_H(\sigma_n)$ denote the posterior probability of the bad state of nature at subdate n , conditional on the information available at that subdate. Let

$$\mathfrak{I}(\sigma_n) = E_n \left\{ \left[\frac{f_H(\sigma_{n+1}) - f_H(\sigma_n)}{f_H(\sigma_n)} \right]^2 \middle| \sigma_n \right\} \quad (39)$$

denote the relative variance of the posterior probability. $\mathfrak{I}(\sigma_n)$ is a measure of the informativeness of the signal accruing at subdate $n + 1$.

Note, from (38), that the ratio formula (25) continues to apply with $q_k(\sigma_n)$ in place of q_k . So at subdate n and for any two assets k and ℓ ,

$$\frac{q_k(\sigma_n) - 1}{q_\ell(\sigma_n) - 1} = \frac{\theta_k(\omega_H)}{\theta_\ell(\omega_H)} \equiv \lambda_{k,\ell}. \quad (40)$$

Under the (strong) assumption of just one liquidity-constrained state, all assets with constant expected dividend are priced according to a linear formula involving the liquidity premium on the generalized fixed income security.

Let V_k and v_k denote the price and return volatilities of asset k conditional on information σ_n :

$$V_k(\sigma_n) = E_n [[q_k(\sigma_{n+1}) - q_k(\sigma_n)]^2 | \sigma_n] \quad (41)$$

and

$$v_k(\sigma_n) = E_n \left[\left[\frac{q_k(\sigma_{n+1}) - q_k(\sigma_n)}{q_k(\sigma_n)} \right]^2 \middle| \sigma_n \right]. \quad (42)$$

With only one liquidity-constrained state, the price of an asset k with constant expected dividend is in state of information σ_n , as we have seen,

$$q_k(\sigma_n) - 1 = f_H(\sigma_n)\theta_k(\omega_H)m_H. \quad (43)$$

This immediately yields

$$V_k(\sigma_n) = \mathfrak{I}(\sigma_n)[q_k(\sigma_n) - 1]^2, \quad (44)$$

$$v_k(\sigma_n) = \mathfrak{I}(\sigma_n) \left[\frac{q_k(\sigma_n) - 1}{q_k(\sigma_n)} \right]^2, \quad (45)$$

$$V_k(\sigma_n) = \lambda_{k,\ell}^2 V_\ell(\sigma_n), \quad (46)$$

$$v_k(\sigma_n) = \lambda_{k,\ell}^2 \left(\frac{q_\ell(\sigma_n)}{q_k(\sigma_n)} \right)^2 v_\ell(\sigma_n). \quad (47)$$

Equations (44) and (45) state that, in absolute as well as relative terms, the volatility of an asset's price is proportional to the square of the asset's liquidity premium. In particular, the asset's price volatility V is increasing and its return volatility v is decreasing as a function of the liquidity premium.²³ Moreover, if price (or return) volatility is expected to be high at date n , this is likely to be the case also at date $n + 1$: Volatility will show persistence over time, because prices follow a martingale. Such persistence can induce serially correlated price and return volatility.²⁴

Equation (46) states that the ratio of the price volatility of two assets is constant over time. Price volatilities move together, because they are driven by the same news concerning the likelihood of a liquidity shortage. The ratio of return volatilities will do the same if the price ratio does not move much.

²³ To be precise, these statements hold true in a region where the informativeness term \mathfrak{I} is constant or moves slowly relative to the liquidity premium. With two states, this informativeness measure is generally state dependent. To illustrate a case with constant informativeness, suppose that information accrues according to a Bernoulli process: At each subdate n , with probability $\pi \in (0, 1)$, the market learns that the economy will not be in the bad state of nature ($f_H(\sigma_n) = 0$). If that happens, then the economy becomes an "Arrow-Debreu economy," in which assets command no liquidity premium and hence $q_k(\sigma_m) = 1$ for all $m \geq n$. In this absorbing state, the informativeness may be taken equal to 0. With probability $1 - \pi$, the economy remains an "LAPM economy" and $f_H(\sigma_n) = (1 - \pi)^{N-n}$. A simple computation shows that $\mathfrak{I}(\sigma_n) = \pi/(1 - \pi)$.

In this example, there is price volatility as long as the liquidity premium is strictly positive. If news accrues that the economy will be replete with liquidity, the liquidity premium goes to zero as will the volatility of prices for assets with constant expected payoffs.

²⁴ Serial correlation requires additional assumptions on the innovation series x_n .

IV. The Yield Curve

A. *The Slope of the Yield Curve and Price Risk*

The theoretical and econometric research on the term structure of interest rates traditionally views the corporate sector as a veil in that asset liability management (ALM) does not impact the yield curve. Many believe, however, that corporate liquidity demand affects the term structure. First, while debt markets are segmented, there is enough substitutability across maturities to induce long and short rates to move up and down together (Culbertson (1957)). Duration analysis, stripping and related financial engineering and innovation activities aimed at tailoring securities to particular investor groups, provide indirect evidence of the value of segmentation. A number of factors such as fiscal incentives, the growth of pension funds, new accounting and prudential rules for intermediaries, and the leverage of the real and financial sectors are likely to affect in different ways the demand for maturities, thereby influencing the term structure. Second, the maturity structure of government debt seems to play a role in the determination of the term structure, a fact that is not accounted for in Ricardian consumption-based asset pricing models.

The yield curve is most commonly upward sloping, although it may occasionally be hump-shaped or inverted over the whole range, or even have an inverted hump-shape (see Stigum (1990), Campbell (1995), and Campbell, Lo, and MacKinlay (1996)).²⁵ It is often argued that an upward-sloping yield curve reflects the riskiness of longer maturities. Investors, so the story goes, demand a price discount as compensation for this risk (which presumably is correlated with consumption if the standard model applies). This argument is based on an analogy with CAPM. It would be worthwhile, though, to provide a precise definition of the notion of “price risk.” CAPM is about the coupon risk of assets. Coupon risk relates to uncertainty about dividends, or, more generally (to encompass uncertainty about preferences and endowments), to uncertainty about the marginal utility of dividends.

Price risk may stem from coupon risk, but it need not. Consider an intertemporal Arrow–Debreu endowment economy (as, say, in Lucas (1978)). In this economy, early release of information about future endowments is irrelevant in that it affects neither the real allocation nor the date-0 price of claims on future endowments. On the other hand, release of information affects asset prices, inducing price risk. In an Arrow–Debreu endowment economy, the date-0 price of claims on date-2 endowments can be entirely unrelated to the variance of their date-1 price (or to the covariance of price and some measure of aggregate uncertainty). Price risk per se is not an aggregate risk and thus need not affect asset prices.

²⁵ There are a number of other stylized facts: short yields move more than long yields; long-term bond prices are highly volatile; and high-yield spreads tend to precede decreases in long rates. Also, the yield curve tends to be flatter when money is tight. An outstanding puzzle is the significant yield differential between one-month T-bills and six-month T-bills (the “term premium puzzle”).

Returning to the yield curve, Treasury bonds are basically default-free. Uncertainty about the rate of inflation, however, creates a coupon risk (for nominal bonds), which, in turn, affects prices. Inflation uncertainty clearly plays an important part in explaining the price risk of long-term bonds. But for short-term bonds the connection is less obvious. A bond that matures in less than a year is quite insensitive to inflation, at least directly. Indirectly, swings in the price of long-term bonds will of course influence short-term prices as long as maturities are partially substitutable.

Our point is to caution against drawing hasty conclusions about the link between price risk and the slope of the yield curve. A theoretical justification based on the Arrow–Debreu model cannot be provided, because in a complete market, price risk stemming from early information release will not carry any risk premium. This opens the door for alternative theoretical approaches to analyzing the yield curve. Our liquidity-based asset pricing model offers one possibility.

B. Long-term Bonds and the Hirshleifer Effect

To obtain some preliminary insights into the effects of liquidity on the term structure, let us again return to the example of Section I except that the noncorporate financial asset is now a Treasury bond, rather than land.²⁶ Assume that the government at date 0 issues two types of bonds: $\bar{\ell}$ short-term bonds yielding one unit of the good at date 1, and \bar{L} (zero coupon) long-term bonds yielding θ units of the good at date 2. We allow for a coupon risk on long-term bonds, so let θ be a random variable with support $[0, \infty)$, density $h(\theta)$, cumulative distribution $H(\theta)$, and mean $E(\theta) = 1$. As discussed above, θ can be interpreted as the date-2 price of money in terms of the good. The case of a deterministic inflation rate (which can be normalized to 0) corresponds to a spike in the distribution at $\theta = 1$. We let q and Q denote the date-0 prices of short- and long-term bonds. A short-term “risk premium” corresponds to $q > Q$. Treasury bonds are the only noncorporate, liquid assets in the economy.

As in Section I, the date-1 income x is assumed to be perfectly correlated across firms, and $g(x)$ and $G(x)$ denote the density and the cumulative distribution of income x . Assume that θ and x are independent. In this economy, firms have no liquidity demand past date 1. This example is meant to illustrate a situation in which most of the liquidity is expected to be employed in the short run.

If there is no coupon risk ($\theta \equiv 1$) or if there is no signal about the realization of θ before date 2 (so that there is a coupon risk, but no price risk at date 1), short-term and long-term bonds will be perfect substitutes and so $q = Q$. Suppose instead that the realization of θ is learned at date 1. Now the

²⁶ As mentioned before, we assume that Treasury bonds are backed up by government property such as land to side-step tax issues.

price at which long-term bonds can be disposed of at date 1, namely θ , will vary. The coupon risk in this case induces a price risk.

Let ℓ and L denote the number of short-term and long-term bonds purchased at date 0 by the corporate sector (in equilibrium, $\ell = \bar{\ell}$ if $q > 1$ and $L = \bar{L}$ if $Q > 1$). The corporate sector solves

$$\max_{\{y(\cdot), \ell, L\}} E_0 \left[by(x) - \frac{(y(x))^2}{2} \right] \quad (48)$$

s.t.

$$E[x - I - y(x) - (q - 1)\ell - (Q - 1)L] \geq 0, \quad (49)$$

and

$$y(x) \leq x + \ell + \theta L \quad \text{for all } x. \quad (50)$$

Let μ again denote the shadow cost of the investors' break-even constraint and let $y^* = b - \mu$ denote the optimal unconstrained reinvestment level. Because $\ell = \bar{\ell}$ and $L = \bar{L}$ in equilibrium, equilibrium prices are characterized by

$$q - 1 = \int_0^\infty \left[\int_0^{y^* - \bar{\ell} - \theta \bar{L}} \left[\frac{b - (x + \bar{\ell} + \theta \bar{L})}{\mu} - 1 \right] g(x) dx \right] h(\theta) d\theta, \quad (51)$$

$$Q - 1 = \int_0^\infty \theta \left[\int_0^{y^* - \bar{\ell} - \theta \bar{L}} \left[\frac{b - (x + \bar{\ell} + \theta \bar{L})}{\mu} - 1 \right] g(x) dx \right] h(\theta) d\theta. \quad (52)$$

Denote the inside integrals in (51) and (52) by $z(\theta)$. It is easily verified that $z'(\theta) < 0$. Consequently,

$$Q - 1 = E[\theta z(\theta)] < E[\theta]E[z(\theta)] = E[z(\theta)] = q - 1. \quad (53)$$

The result in (53) holds true more generally. As long as the marginal liquidity service $m'(\cdot)$ (represented by the innermost integrand in (51) and (52)) is decreasing in income, we have $z'(\theta) < 0$ and (53) will follow.

In reference to our earlier discussion, it should be stressed that price variation alone is not the problem here. Without a shortage of liquidity, long- and short-term bonds are priced the same irrespective of price uncertainty. Only in the presence of liquidity shortages will price uncertainty cause long-term bonds to sell at a discount relative to short-term bonds. The reason is evident from the derivation of (53). Because variation in the price of the long-term bond is negatively correlated with the marginal value of liquidity, liquidity shortages induce an endogenous degree of risk aversion. This is

quite different from exogenously assuming that consumers or firms are averse to price risk. In the liquidity approach, asset prices reflect a skewness in risk tolerance, which causes changes in the term structure as a function of changes in the likelihood of liquidity shortages.

Note that if no information about θ arrived at date 1, or if there would be no uncertainty about inflation, long-term bonds would again offer the same liquidity service as short-term bonds. This contrasts with an Arrow–Debreu economy, in which early arrival of information never is harmful. Early information has no impact on an endowment economy, but may generate social gains in a production economy due to improved decision making.

Our model features a logic similar to Hirshleifer's (1971) idea that early information arrival may make agents worse off. It differs somewhat from Hirshleifer's, in that in his model, information arrives before entrepreneurs and investors sign a contract. In our model, it is the investors' inability to commit to bringing in funds at date 1 that constrains contracting and makes information leakage problematic. Investors cannot offer insurance against variation in the price of long-term bonds in liquidity-constrained states, because insurance payments can never exceed the value of the available liquid assets in the economy, that is, the value of the very bonds that firms want insured.

B.1. Neutrality of Pure Price Risk

Following Section III, assume that news about the date-1 state of nature accrues between date 0 and date 1, at subdates $n = 1, \dots, N$; that is, at each subdate n , a signal σ_n accrues that is informative about the date-1 income and/or the coupon on the long-term bond. Prices of short- and long-term bonds, $q(\sigma_n)$ and $Q(\sigma_n)$, vary with the news, but this price risk has no impact on the date-0 prices q and Q , which remain given by (51) and (52), since the corporate sector in equilibrium does not reshuffle its portfolio of liquid assets (this is the point made earlier that capital gains and losses on financial assets have offsetting effects). In other words, price risk has no impact on the slope of the date-0 yield curve. On the other hand, news affects the slope of the yield curve at subdates.

B.2. The Yield Curve May Be Upward Sloping Even in the Absence of Coupon Risk

In the absence of any coupon risk, $q = Q$. Let i_1 and i_2 denote the yields on short- and long-bonds at date 0. These yields are negative in our model because the consumers' rate of time preference is normalized to zero. We have

$$q = Q = \frac{1}{1 + i_1} = \frac{1}{(1 + i_2)^2}, \quad (54)$$

and so

$$i_1 = \frac{1}{q} - 1 < i_2 = \frac{1}{\sqrt{Q}} - 1. \quad (55)$$

This well-known example merely makes the point that riskiness of long-term bonds is not a necessary condition for the existence of a term premium. The yield curve can be upward sloping, as here, simply because the corporate sector has no liquidity demand at date 2. This suggests that an upward slope is associated with relatively more pressing short-term liquidity needs, perhaps because the firm has less flexibility to adjust plans in the short term.

B.3. Other Shapes of the Yield Curve

Suppose the income shock x and reinvestment decision y take place at date 2, and the private benefit accrues at date 3. Investments and financing still occur at date 0. In this temporal extension of the model, date 1 is just a “dummy date,” at which nothing happens. Suppose that the government still issues short-term bonds (maturing at date 1) and long-term bonds (maturing at dates 2 and 3). Short-term bonds offer no liquidity service and so $q = 1$. Hence, the short rate (equal to zero) exceeds the long rates, and we obtain an inverted yield curve.

This example makes the simple point that if the corporate sector does not expect to face liquidity needs in the short run, it does not pay a liquidity premium on short-term securities, and so they will yield more than long-term securities.

V. Conclusion

For a long time, corporate finance has been treated as an appendix to asset pricing theory, with CAPM frequently used as the basic model for normative analyses of investment and financing decisions. While standard textbooks still reflect this tradition, the modern agency-theoretic literature is starting to influence the way corporate finance is taught. This paper takes the next logical step, which is to suggest that if financing and investment decisions in firms reflect agency problems—as seems to be widely accepted—then it is likely that modern corporate finance will cause adjustments in asset pricing theories, too.

Our paper is a very preliminary effort to analyze the influence of corporate finance on asset pricing. We have employed a standard agency model in which part of the returns from a firm’s investment cannot be pledged to outsiders, raising a demand for long-term financing, that is, for liquidity. We have also assumed that individuals cannot pledge any of their future income, so that borrowing against human capital is impossible. As a result, the

economy is typically capital constrained, implying that collateralizable assets are in short supply.²⁷ Such assets will command a premium, which is determined by the covariation of the asset's return with the marginal value of liquidity in different states. Risk-neutral firms are willing to pay a premium on assets that help them in states of liquidity shortage. This is a form of risk aversion, but unlike in models based on consumer risk aversion, return variation within states that experience no liquidity shortage is inconsequential for prices. Liquidity premia have a built-in skew.

One consequence of this skew is that price volatility tends to be higher in states of liquidity shortage, as we illustrated in Section III. Another consequence is that long-term bonds, because of a higher price risk, tend to sell at a discount relative to short-term bonds as we showed in Section IV. This may be one reason why the yield curve is most of the time upward sloping, a feature that does not readily come out of standard models of asset pricing.

The price dynamics in our model satisfy standard Euler conditions—in particular, prices follow a martingale as long as there is no readjustment in the corporate sector's coordinated investment plan. It is an interesting possibility that marginal rates of substitution for the corporate sector may be quite different, and perhaps more volatile in the short run, than the marginal rates of substitution of a representative consumer. This could help to resolve some of the empirical difficulties experienced with consumption-based asset pricing models, which appear to feature too little variation in IMRSs.

Our model is quite special in that asset prices are entirely driven by a corporate demand for liquidity; consumers hold no bonds or other assets that sell at a premium. It has been suggested to us that once the model is changed so that consumers also have a liquidity demand, IMRSs of consumers and firms will be equalized, and we are back to the old problem with excess asset price volatility. However, if consumers participate selectively in asset markets, then the IMRSs of the relevant subpopulation may have high volatility and yet be hard to detect. In this case, the equality between consumer and producer IMRSs can be exploited in the reverse: by evaluating corporate IMRSs, we can infer what the IMRS of the representative consumer subpopulation is. This may be a useful empirical strategy if firm data are more readily available and easier to analyze than consumer data.

Finally, we note that violations of the martingale condition, as illustrated by the end-of-period drop in the liquidity premium, may help to explain the well-known paradox that prices of long-term bonds tend to move up rather

²⁷ Our model does not incorporate fiat money. All our financial assets are backed up by claims on real goods. In reality, consumers and firms hold cash, or cash equivalent assets, as buffers against liquidity shocks. Since we do not know of a satisfactory way to introduce fiat money, it is hard to say how fiat money would change our analysis. We think that short-term bonds in the model correspond closely to cash as long as one is not interested in the substitution between short-term bonds and fiat money in the context of monetary policy.

than down, following a period in which the yield spread (long/short) is exceptionally high. This finding is very difficult to reconcile with the standard expectations theory (Campbell (1995)), but could perhaps be accounted for in a theory where liquidity demand shifts between short- and long-term instruments in response to expected fluctuations in liquidity needs.

Appendix: Treatment of the Productive Sector as an Aggregate

Suppose that there are n firms, $i = 1, \dots, n$, each run by an entrepreneur. Firm i starts with initial wealth A_i and invests I_i , so that the net outlay for investors is $N_i(I_i) = I_i - A_i$. At date 1, in state ω , firm i takes a decision $d_i(\omega)$ in a subset $D_i(I_i, \omega, L_i^n(\omega))$ of the technologically feasible set D_i , where $L_i^n(\omega)$ is the net liquidity available to firm i in state ω . $L_i^n(\omega) > 0$ means that firm i uses liquidity, and $L_i^n(\omega) < 0$ means that firm i supplies liquidity in state ω . The decision $d(\omega)$ generates an expected pledgeable income $r_i(I_i, \omega, d_i(\omega))$ and a total expected income $R_i(I_i, \omega, d_i(\omega))$ from productive assets. Let

$$B_i(I_i, \omega, d_i(\omega)) \equiv R_i(I_i, \omega, d_i(\omega)) - r_i(I_i, \omega, d_i(\omega)) \quad (\text{A1})$$

denote the nonpledgeable income of firm i that must go to entrepreneur i . Entrepreneur i may, however, be paid more than B_i . Let $t_i(\omega) \geq 0$ denote the expected transfer on top of the nonpledgeable income B_i ($t_i(\cdot)$ could without loss of generality be chosen equal to zero in the example in Section I). So entrepreneur i obtains, in state ω , $B_i(I_i, \omega, d_i(\omega)) + t_i(\omega)$. If firm i withdraws gross liquidity $t_i(\omega)$ from noncorporate assets, then the net liquidity available to the firm is $L_i^n(\omega) = \tilde{L}_i(\omega) - t_i(\omega)$.

The economically feasible set for firm i reflects the fact that no investor wants to accept negative NPV investments at date 1:

$$D_i(I_i, \omega, L_i(\omega) - t_i(\omega)) = \{d_i \in D_i \mid r_i(I_i, \omega, d_i(\omega)) - t_i(\omega) + \tilde{L}_i(\omega) \geq 0\}. \quad (\text{A2})$$

Let

$$D(I, \omega, L^n(\omega)) \equiv \left\{ \chi_{i=1}^n D_i(I_i, \omega, L_i^n(\omega)) \mid \sum_{i=1}^n L_i^n(\omega) \geq L^n(\omega) \right\} \quad (\text{A3})$$

denote the product decision set, with generic element $d(\omega)$. This set is the economically feasible set of the corporate sector in state ω , when liquidity can be freely redistributed across firms. With a slight abuse of notation, I here is the vector of firm investments (I_1, \dots, I_n) .

According to Assumption 1, each firm i can be seen as buying state-contingent liquidity $\tilde{L}_i(\omega)$ at a market determined price $m(\omega)$. Firm i therefore solves

$$\max_{\{I_i, \tilde{L}_i(\cdot), d_i(\cdot), t_i(\cdot)\}} \{E_0[B_i(I_i, \omega, d_i(\omega)) + t_i(\omega) - A_i]\} \quad (\text{A4})$$

s.t.

$$E_0[r_i(I_i, \omega, d_i(\omega)) - t_i(\omega)] \geq I_i - A_i + E_0[m(\omega)\tilde{L}_i(\omega)] \quad (\text{A5})$$

and

$$d_i(\omega) \in D_i(I_i, \omega, \tilde{L}_i(\omega) - t_i(\omega)). \quad (\text{A6})$$

Letting $1/w_i$ denote the shadow price of the budget constraint (which is strictly positive), this program, under our concavity assumptions, is equivalent to

$$\begin{aligned} \max_{\{I_i, \tilde{L}_i(\cdot), d_i(\cdot), t_i(\cdot)\}} & E_0[w_i(B_i(I_i, \omega, d_i(\omega)) + t_i(\omega)) \\ & + r_i(I_i, \omega, d_i(\omega)) - t_i(\omega) - I_i + A_i - m(\omega)\tilde{L}_i(\omega) - A_i] \end{aligned} \quad (\text{A7})$$

s.t.

$$d_i(\omega) \in D_i(I_i, \omega, \tilde{L}_i(\omega) - t_i(\omega)) \quad \text{for all } \omega, \quad i = 1, \dots, n. \quad (\text{A8})$$

Summing over the individual programs, we get

$$\max E_0 \left[\sum_i w_i [B_i + t_i] + \sum_i [r_i - t_i - I_i + A_i] - m(\omega) \sum_i \tilde{L}_i \right] \quad (\text{A9})$$

s.t.

$$d_i(\omega) \in D_i(I_i, \omega, \tilde{L}_i(\omega) - t_i(\omega)) \quad \text{for all } \omega, i = 1, \dots, n. \quad (\text{A10})$$

It follows that the market equilibrium solves

$$\max_{\{I, L, d(\cdot), t(\cdot)\}} E_0 \left[\sum_i w_i [B_i(I_i, \omega, d_i(\omega)) + t_i(\omega)] \right] \quad (\text{A11})$$

s.t.

$$E_0 \left[\sum_i [r_i(I_i, \omega, d_i(\omega)) - t_i(\omega)] \right] \geq \sum_i (I_i - A_i) + E_0 \left[m(\omega) \sum_i \tilde{L}_i(\omega) \right] \quad (\text{A12})$$

and

$$d(\omega) \in D \left(I, \omega, \sum_k \theta_k(\omega) L_k - \sum_i t_i(\omega) \right). \quad (\text{A13})$$

In this last step, we have replaced the individual firms' economically feasible set with the corporate sector's economically feasible set. This is permissible, because the corporate sector's budget constraint is unaffected by how liquidity is distributed among individual firms in each state ω .

Let μ denote the multiplier of the break-even constraint in the corporate program above. The first-order condition with respect to $t_i(\omega)$ is

$$\begin{aligned} \text{Either } t_i(\omega) = 0 \quad \text{and} \quad w_i &\leq \mu[1 + m(\omega)] \\ \text{or } t_i(\omega) > 0 \quad \text{and} \quad w_i &= \mu[1 + m(\omega)]. \end{aligned} \quad (\text{A14})$$

Now assume that there exists at least one state of nature with excess liquidity (this is the case in all our examples). Then for all i , $t_i(\omega) = 0$ whenever $m(\omega) > 0$. That is, pledgeable income is never distributed to entrepreneurs in states of liquidity shortage. Thus the equilibrium is as described in the text, with $B = \sum_i w_i B_i$, and $r = \sum_i r_i$.

Remarks:

1. We did not consider individual rationality constraints for entrepreneurs because such constraints only affect the set of weights w_i .
2. The weights $\{w_i\}$ are endogenous, so one cannot conduct comparative statics exercises with this representation, and we do not. However, the analysis of price dynamics in Section III and of the yield curve in Section IV is valid, because it entails no rebalancing of portfolios.

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