Costly Voting
Author(s): Tilman Börgers
Source: *The American Economic Review*, Vol. 94, No. 1 (Mar., 2004), pp. 57-66
Published by: American Economic Association
Stable URL: http://www.jstor.org/stable/3592769
Accessed: 21/08/2009 18:20

Costly Voting

By Tilman Börgers*

What are good voting rules if voting is costly? We analyze this question for the case that an electorate chooses among two alternatives. In a symmetric private value model of voting we show that majority voting with voluntary participation Pareto-dominates majority voting with compulsory participation as well as random decision-making. (*JEL* C70, D72)

Should participation in votes be voluntary or should it be compulsory? How much pressure should be exerted on individuals to participate in votes? These are the questions which this paper addresses. The analysis of this paper sheds light on the way in which companies, clubs, or academic departments should organize meetings and votes.

The effects of voluntary participation in votes have been the subject of some recent empirical literature on voting. Matthew Turner and Quinn Weninger (2001) find for a particular industry (Mid-Atlantic surf clam and ocean quahog fishery) that firms which prefer moderate policies are less likely to participate in public meetings with voluntary participation than firms which prefer extreme policies. George Bulkley et al. (2001) have investigated the U.K.'s House of Lords in which, incidentally, participation is financially rewarded. They find that members of the House of Lords are less likely to participate in votes if they are not affiliated with a party than if they are. Voluntary participation in votes thus induces self-selection among voters. The question arises whether this selection is efficient in some sense. This is the question which is formalized in this paper.

The policy problem which we analyze arises also in national elections. While in most countries participation in such elections is voluntary, some countries (e.g., Belgium, Italy) have tried to make it compulsory. We are cautious about the relevance of our results in this context, though. Our analysis is built on game-theoretic models of voting in which participation decisions are rational, and are driven by the probability that an individual's vote is pivotal. With large electorates this probability is, under most voting rules, close to zero, yet empirically observed participation rates are often high. This is *The Paradox of Voting* (Anthony Downs, 1957; John Ferejohn and Morris Fiorina, 1974). This paradox suggests that a conventional, game-theoretic analysis of costly voting is out of place if large electorates are considered. By contrast, for small electorates there seems no reason why observed voting behavior should not be rational. This is why our paper is meant for small electorates only.

Our main finding is that voluntary majority voting is superior to compulsory voting. The intuition for this result is simple. There is a negative externality arising from voting. Any individual's vote makes it less likely that other voters are pivotal. This reduces other voters' expected utility. When deciding whether to vote, individuals take account of the benefit of a vote to themselves, but ignore the negative externalities which they cause for others. Thus, individual incentives to participate in a voluntary vote are too large rather than too small. Making voting compulsory, or exerting pressure on individuals to participate, moves the system into the wrong direction, and makes things worse.

The finding that equilibrium participation in votes is too high may appear at first surprising. Public debate, after all, seems more concerned with too low rather than too high participation. Our finding becomes more intuitive if one considers analogous contexts. For example, the average length of contributions to discussions in

* Department of Economics, University College London, Gower Street, London WC1E 6BT, United Kingdom (e-mail: t.borgers@ucl.ac.uk). I would like to thank Tim Feddersen and an anonymous referee for their comments. I am grateful to the Economic and Social Research Council (U.K.) for financial support through the Centre for Economic Learning and Social Evolution.

department meetings seems excessive. Similarly, people probably spend an excessive amount of time intriguing to influence collective decisions. Our finding of excessive participation in votes is a similar instance of overinvestment into political activities.

Although we thus argue that the effect on which this paper is based is intuitively plausible, the purpose of the paper is, of course, not to make a general claim that in practice participation rates are always too high. Rather, we construct a model that isolates one particular effect which hasn't received enough attention. The model should be seen as a benchmark model, and more general models throw up other effects which work into the opposite direction of the effect which we study here.

In our model there are only two possible collective choices, for example two candidates. We thus avoid the complications of the well-known Condorcet paradox which arises if there are three or more alternatives. We postulate positive voting costs, which are privately observed. Full participation by all voters optimizes in some sense the "quality" of the collective decision, but it also maximizes the costs of the vote. A random decision without vote minimizes the quality of the collective decision, but it also minimizes the voting costs. Our main result shows that voting with voluntary participation is superior both to enforced participation and to random decision making without vote.

Our analysis is set in a private value model of voting where preferences reflect idiosyncratic individual tastes. In a common value model of voting (for example, Timothy Feddersen and Wolfgang Pesendorfer, 1996), where individuals have identical tastes but different information, there will be positive externalities to voting which can mitigate or outweigh the negative externality which we identify. The positive externality is that those who vote improve for everybody the information on which the collective decision is based. Indeed, responding to the first version of this paper, Sayantan Ghosal and Ben Lockwood (2003) have displayed a common value model in which the negative externality on which this paper focuses is present, but is locally outweighed by the positive informational externality. This means that in their model, starting from equilibrium participation rates, a small increase in participation is always welfare improving. As the proof of our main result shows, the opposite is the case in our model. Ghosal and Lockwood go on to show that in some cases they can even obtain the opposite of our main result, i.e., that compulsory voting can dominate voluntary voting. They also extend their results to the case that there is a private value component in voters' utility, provided that the relative weight of the private value component is not too large. Conversely, it is easy to show that our results continue to hold if the relative weight of the private value component is sufficiently large.

Our model is symmetric, both with respect to alternatives, and with respect to individuals. It seems natural to consider the design of voting rules in a setting in which there are no *ex ante* built-in differences between alternatives or between individuals. However, the symmetry assumption holds in our model also at the *interim* stage, i.e., after individuals have received their private information, and before they vote. This means that even at this interim stage individuals think that each alternative is equally likely to be favored by each of the other individuals. Jacob Goeree and Jens Grosser (2003) have recently studied a model in which preferences are correlated, and therefore Bayesian updating leads individuals to believe that everybody else is more likely to share their own preferences than otherwise. The negative externality identified in this paper remains present in their model, but an additional positive externality is introduced. Those who do not vote benefit on average from the participation of those who vote, because they are likely to share their preferences. As a consequence, if the correlation of preferences is sufficiently strong, a small increase in the equilibrium level of participation is beneficial. Goeree and Grosser do not discuss whether the effect is so strong that compulsory voting may be optimal. If the correlation in Goeree and Grosser's model is sufficiently low, then our results continue to be true.[1]

[1] Colin Campbell (1999) studies a related model, but, unlike Goeree and Grosser, he allows the distribution of the costs of voting (or, equivalently, of the strength of preference for one alternative rather than another) to depend on the alternative which a voter favors. He constructs conditions under which in sufficiently large populations the alternative that is *less* likely to be preferred by voters is *more* likely to win a majority vote. He points out that this is undesirable from the point of view of economic welfare.

Beyond what has been explained so far, our paper also makes a technical contribution. We prove the uniqueness of symmetric equilibrium in a model of costly voting. This result, although not very complicated, has, to our knowledge, not been obtained in earlier papers. Goeree and Grosser (2003) extend the result to their setting. Ghosal and Lockwood (2003) show that uniqueness does not hold in the presence of common values.

Because in our model, and in the alternative frameworks discussed above, participation decisions are typically not optimal, an interesting problem of designing the optimal voting system arises. The discussion paper version of this paper[2] analyzes for the benchmark setup of the current paper whether there are voting systems which are superior to voluntary majority voting. The three systems which we consider are voting in a committee, voting over a status quo, and sequential voting. For each of these three systems the discussion paper version establishes that they are superior to voluntary majority voting for certain distributions of voters' characteristics. For committee voting, and status quo voting, we also identify distributions for which voluntary majority voting is superior.

Beyond the papers already mentioned, studies that are related to ours include early work by Thomas R. Palfrey and Howard Rosenthal (1983, 1985) and by John O. Ledyard (1981, 1984). Palfrey and Rosenthal (1983) assume that the numbers of voters favoring each of two alternatives is commonly known, and that the voters' voting costs are identical and commonly known. They find multiple equilibria, including some in which the minority is more likely to win than the majority. They note the inefficiency of such equilibria. Palfrey and Rosenthal (1985) extend the earlier paper to the case of incomplete information about voting costs, but give no welfare analysis.[3]

The main result of Ledyard's two papers is Theorem 1 in Ledyard (1984), which shows the welfare optimality of equilibrium voting under voluntary majority voting. Ledyard's model differs from ours in that he endogenizes in a Downs (1957) type model the two alternatives voters can choose from. The optimality result applies to an equilibrium in which candidates choose identical positions which maximize voters' *ex ante* welfare, and in which nobody votes. Because we do not endogenize the candidates' platforms we are looking, in a sense, for a stronger optimality property than Ledyard does.[4]

Martin Osborne et al. (2000) have a model of costly participation in a collective decision process where the set of alternatives is some convex subset of some Euclidean space. They do not model explicitly how the collective decision is arrived at, but instead work with a reduced-form "compromise function" which describes the collective decision as a function of the positions of all participating individuals, for example, the median. Our question of how voting rules affect participation decisions corresponds to the question of how different compromise functions affect participation. Osborne et al., however, do not focus on this question. Their main interest is in the features of equilibria for given compromise functions.[5]

I. Setup

There are n individuals: $i = 1, 2, \dots, n$. To avoid trivial case distinctions, we assume: $n \geq 3$. The individuals form a club which has to choose one of two alternatives: $a = A, B$. This is a collective choice problem. One alternative must be chosen, and this alternative will apply to all members of the club. An example would be that the club has to select either A or B as its new chairman.

The relevant characteristics of an individual are summarized in that individual's "type" $t_i = (a_i, c_i) \in \{A, B\} \times \Re_+$ where the first component, a_i, is the alternative which individual i favors,[6] and the second component,

[2] Available from the author upon request.

[3] The case of incomplete information about preferences is mentioned in Palfrey and Rosenthal (1985), but the only result provided focuses on participation rates in large electorates. Palfrey and Rosenthal's work is a precursor of Campbell's (1999) paper that we cited in footnote 2.

[4] Ledyard's result is also built on an assumption of "many" voters. For reasons explained above we are reluctant to focus on this case.

[5] Bulkley et al. (2001) analyze a model that is closely related to that of Osborne et al. (2000).

[6] We rule out the possibility that individuals are indifferent between the two candidates. If voting is costly, such individuals will never vote. Therefore, they can safely be omitted from the analysis.

c_i, indicates individual i's costs of participating in a collective decision process. If individual i is of type $t_i = (a_i, c_i)$, then i's von Neumann Morgenstern utility is highest if i's most favored alternative, a_i, is chosen, but individual i does not participate in the decision-making process. In that case, individual i's utility is normalized to be equal to 1. If the alternative which i ranks second is chosen, and i does not participate, then i's utility is equal to 0. Now consider the utility of individual i if i *does* participate in the decision-making process. In this case we simply subtract from the utilities described so far individual i's participation costs c_i. Hence, if a_i is chosen and individual i does vote, then her utility is $1 - c_i$, and if a_i is not chosen, and i does vote, then her utility is $-c_i$.

Note that we are assuming that the costs of participation are independent of whether individual i's participation is compulsory or voluntary. The costs are also independent of the decision-making mechanism which society uses, of the strategy which individual i chooses in that mechanism, and of the alternative which society chooses.[7] These assumptions are made for simplicity.

Each individual's type t_i is a random variable. The two components of any individual's type, a_i and c_i, are stochastically independent of each other. For any individual i the alternative a_i, which individual i favors, is with probability ½ equal to A, and with probability ½ equal to B. The participation costs c_i have a distribution function F which is the same for all individuals, and which has support $[\underline{c}, \bar{c}]$ where $0 \leq \underline{c} < \bar{c}$. The distribution function has a density f which is positive on all of the support.

Note that the previous paragraph contains two distinct symmetry assumptions. Firstly, our model is symmetric with respect to alternatives. This means two things: For each individual the probability that he favors any given alternative is the same for both alternatives. Moreover, the conditional distribution of participation costs is the same for both alternatives. Secondly, our model is symmetric with respect to individuals. For each individual, the distribution of types is the same. As mentioned in the introduction, the most important part of our symmetry assumption is that the symmetry regarding individuals' preferences for each of the two alternative is maintained at the *interim* stage. Goeree and Grosser (2003) investigate alternative assumptions.

Next we assume that the type t_i of individual i is stochastically independent of the type t_j of individual $j \neq i$. This has two important implications. Firstly, it means that we are considering a *private* value model of voting rather than a *common* or *affiliated* value model. In a private value model of voting, types reflect purely private tastes. In a common value model, by contrast, all voters would agree on which candidate is best if they all had the same information, and differences of opinion result only from the fact that different individuals hold different pieces of information. The importance of the private value assumption to our analysis was already explained in the introduction.

The second implication of the independence assumption for types is that different individuals' participation costs are not correlated. If they were, then an individual who found that his or her participation costs were low (for example, because the weather is bad, and therefore the opportunity costs of voting are low) would deduce that other individuals' participation costs were also low (because the weather is the same for everybody), and that therefore it would be less likely that any individual vote (or other action) matters. Such countervailing incentives make the analysis of rational participation decisions much more complicated (Michael Landsberger and Boris Tsirelon, 1999, 2000).

Finally, we assume that individual i observes his own type t_i, but not the type of any other individual. This, too, should be thought of as a benchmark assumption. If we removed this assumption, we would move into the direction of the model of Palfrey and Rosenthal (1983) that was mentioned in the introduction.

Our comparison of different decision-making mechanisms will be based on individuals' expected utilities in the equilibrium outcomes of these mechanisms. We shall analyze Bayesian equilibria of the mechanisms which we propose, and then calculate each individual's expected utility, assuming that these equilibria are played. We shall deal with the special problem of mechanisms with multiple equilibria on a case by case basis. We shall calculate individ-

[7] Costs would depend on the alternative which society chooses if, for example, agent i finds it more painful to participate in a majority vote and to be on the losing side, than to participate and to be on the winning side.

uals' expected utility on an *ex ante* basis. By this we mean that we calculate expected utility assuming that individuals' preferences over alternatives, and individuals' participation costs, have not yet been determined.

The symmetric nature of our model will allow us to base our comparison of different decision-making mechanisms exclusively on *Pareto* comparisons. We shall say that one mechanism Pareto-dominates another mechanism if all individuals' *ex ante* expected utility in a Bayesian equilibrium of the former mechanism is higher than it is in a Bayesian equilibrium of the latter mechanism.

Because of the additive nature of preferences in our model one can decompose welfare comparisons into two components. Individuals care firstly about the quality of the collective decision, i.e., about the probability with which this decision is the alternative which they prefer. They care secondly about the cost at which the decision is reached, i.e., the expected value of their participation costs. There is a sense in which individuals agree *ex ante,* before types are determined, about what the collective decision rule should be. Consider any collective choice rule which assigns to profiles $(a_1, \ldots, a_n)$ of preferred alternatives a probability distribution over A and B. Suppose that the rule is symmetric with respect to individuals. Neglect, for the moment, the costs of decision making. Then *ex ante* all individuals strictly prefer the rule which always picks the alternative favored by the majority over all other rules. This is easy to see. The intuitive reason is that *ex ante* everybody is more likely to be a member of the majority than of the minority.

Now perfect decision making in this sense can be achieved by majority voting if everybody is forced to vote. We shall take this mechanism as our starting point in the next section. The focus of the paper is then on the extent to which one might be willing to accept a lower quality of collective decision making in return for lower costs of the collective decision process.

II. Comparison of Voting Rules

In this section we shall compare three mechanisms for collective decision making: *Compulsory Majority Voting, Random Decision Making,* and *Voluntary Majority Voting.* We begin with *Compulsory Majority Voting.* Under *Compulsory Majority Voting* each individual is forced to participate. Individuals have to vote for either A or B. The alternative that receives the majority of votes is selected. If the two alternatives receive exactly the same number of votes, each is selected with probability ½.

This mechanism has multiple Bayesian equilibria. One type of equilibrium is that all individuals vote for the same alternative a, independent of their personal preferences. This is an equilibrium because, if all individuals vote for the same alternative a, then no single vote affects the majority,[8] and therefore any vote is optimal. However, voting against one's true preferences is obviously a *weakly* dominated strategy. Although there are some arguments in defense of weakly dominated strategies, we shall simplify our analysis by assuming that weakly dominated strategies will not be played. We are then left with a single equilibrium of *Compulsory Majority Voting:* every individual votes for his most preferred alternative. Thus, *Compulsory Majority Voting* achieves perfect decisions, albeit at the expense of maximal participation costs.

Random Decision Making is at the other extreme of symmetric mechanisms. It is the mechanism in which no individual is invited, nor indeed allowed, to participate in the decision making. Each of the two alternatives is selected with probability ½. Thus, the quality of decision making under this mechanism is low. On the other hand, no participation costs arise.

Voluntary Majority Voting is between the two extreme mechanisms discussed so far. Under *Voluntary Majority Voting* each individual can choose whether to participate in the vote. If an individual chooses to participate, he can vote for A or B. The alternative with the larger number of votes is selected. If both alternatives get exactly the same number of votes, each is selected with probability ½.

Under this mechanism, voting against one's true preference is a *strictly* dominated strategy. The dominating strategy is not to participate. For the analysis of Bayesian equilibria it thus suffices to consider an individual's choice between not participating, and voting for the individual's true preference. We restrict attention to symmetric Bayesian equilibria. By this we

[8] Recall that we have assumed $n \geq 3$.

mean Bayesian equilibria which satisfy two distinct symmetry conditions. The first is that an individual's participation decision depends only on the individual's participation costs, and not on the candidate whom the individual favors. The second condition is that all individuals choose the same strategy. We are thus looking for a *voting strategy* of the form: $s : [\underline{c}, \bar{c}] \rightarrow \{0, 1\}$ which is the same for all individuals, and where $s_i(c_i) = 0$ (resp. 1) means that an individual i does not vote (resp. does vote) if her costs of voting are c_i.

All individuals choosing voting strategy s is a Bayesian equilibrium if and only if for almost all values of c_i the decision $s(c_i)$ maximizes individual i's expected utility given that all other individuals play s. When discussing equilibrium strategies, we shall ignore sets of possible cost values which are of measure zero. So, for example, we shall call an equilibrium "unique" if all equilibria are identical to this equilibrium except possibly for a set of cost values which is of measure zero.

In equilibrium, individual i will vote if the expected benefits of voting are larger than the costs of voting c_i. The expected benefits of voting for individual i, assuming that all other individuals play voting strategy s, don't depend on the details of s. Rather, they only depend on the *ex ante* probability, *before* learning c_i, with which any individual votes. This probability is: $p \equiv \int_{\underline{c}}^{\bar{c}} s(c)f(c)\,dc$.

Consider an individual with given and fixed preference for alternative a. The expected benefits of voting to individual i are $B(p) = \frac{1}{2}\Pi(p)$. Here, $\Pi(p)$ is the probability that the difference between the votes for i's preferred alternative a, and the votes for the other alternative, is -1 or 0. In these two cases, the voter's vote makes a difference to the outcome, and voter i is *pivotal.* If i is pivotal, the effect of his vote is to increase his expected utility by exactly $\frac{1}{2}$. This is because he either turns a loss into a draw, or a draw into a win. Hence he increases the probability that his preferred alternative is chosen by $\frac{1}{2}$. Since his utility is 1 if his preferred alternative is chosen, and 0 otherwise, the expected benefit from a pivotal vote is $\frac{1}{2}$.

It remains to investigate $\Pi(p)$. Note that $\Pi(0) = 1$. Our further results regarding Π are summarized in the following remark. Observe that the properties of $\Pi(p)$ immediately carry over to $B(p)$ because $B(p) = \frac{1}{2}\Pi(p)$.

Remark 1: $\Pi(p)$ is a differentiable function of p, and $\Pi'(p) < 0$ for all $p \in (0, 1)$.

PROOF:

Denote by $\tilde{\ell}$ the number of individuals other than i who choose to vote. Thus $\tilde{\ell}$ is a random variable with binomial distribution with parameters $n - 1$ and p. The probability that $\tilde{\ell}$ takes any particular value ℓ is given by:

$$\binom{n-1}{\ell} p^{\ell}(1-p)^{n-1-\ell}.$$

Note that an increase in p leads to a rightward shift in the sense of first-order stochastic dominance of the distribution of $\tilde{\ell}$.

Conditional on the number of voters being ℓ we need to calculate the probability that voter i is pivotal. We shall denote this probability by $\pi(\ell)$. We begin by noting that $\pi(0) = 1$ and $\pi(1) = \frac{1}{2}$. The latter is true because with probability $\frac{1}{2}$ the other voter votes for the alternative which i regards as inferior, in which case i is pivotal, and with probability $\frac{1}{2}$ she votes for i's preferred alternative, in which case i is not pivotal.

In general, if $\ell \geq 1$ and ℓ is odd, then voter i is pivotal if his preferred alternative receives $\frac{\ell - 1}{2}$ votes whereas the other alternative receives $\frac{\ell + 1}{2}$ votes. This occurs with probability:

$$(1) \quad \pi(\ell) = \binom{\ell}{\frac{\ell-1}{2}} \left(\frac{1}{2}\right)^{(\ell-1)/2} \left(\frac{1}{2}\right)^{(\ell+1)/2} = \binom{\ell}{\frac{\ell-1}{2}} \left(\frac{1}{2}\right)^{\ell}.$$

Now suppose again that $\ell \geq 1$ and that ℓ is odd, and consider the case that $\ell + 1$ individuals vote.[9] Then voter i is pivotal if the number of votes for his preferred alternative is $\frac{\ell + 1}{2}$. This occurs with probability:

[9] Assume that $\ell \leq n - 2$.

$$(2)\quad \pi(\ell+1) = \binom{\ell+1}{\frac{\ell+1}{2}}\left(\frac{1}{2}\right)^{(\ell+1)/2}\left(\frac{1}{2}\right)^{(\ell+1)/2}$$

$$= \binom{\ell}{\frac{\ell-1}{2}}\frac{\ell+1}{(\ell+1)/2}\left(\frac{1}{2}\right)^{\ell+1}$$

$$= \binom{\ell}{\frac{\ell-1}{2}}\left(\frac{1}{2}\right)^{\ell} = \pi(\ell).$$

Next suppose that, still for the same ℓ, the number of individuals who vote is $\ell + 2$.[10] Since the number of voters is then again odd, we can use formula (1) to conclude that the probability of voter i being pivotal is:

$$(3)\quad \pi(\ell+2) = \binom{\ell+2}{\frac{\ell+1}{2}}\left(\frac{1}{2}\right)^{\ell+2}$$

$$= \frac{\ell+2}{\ell+3}\binom{\ell}{\frac{\ell-1}{2}}\left(\frac{1}{2}\right)^{\ell}$$

$$= \frac{\ell+2}{\ell+3}\,\pi(\ell+1)$$

Note that the formula in (3) gives a strictly smaller value than formulas (1) and (2).

The three formulas (1), (2), and (3) together show how the probability of voter i being pivotal depends on ℓ. Recall that for $\ell = 0$ the probability is 1 and for $\ell = 1$ the probability is ½. As we increase ℓ further, if we move from an odd to an adjacent even number of voters, the probability of being pivotal goes down. If we move from an even to an adjacent odd number of voters, the probability of being pivotal stays the same.

We can now prove the remark. We begin by noting that:

$$\Pi(p) = \sum_{\ell=0}^{n-1}\binom{n-1}{\ell}p^{\ell}(1-p)^{n-1-\ell}\pi(\ell).$$

[10] Assume that $\ell \leq n - 3$.

Π is differentiable because it is polynomial. The easiest way to see that its derivative is strictly negative is to see that raising p leads to a right shift in first-order stochastic dominance in the distribution of ℓ. Moreover, as described above, the conditional probability that voter i is pivotal, conditional on ℓ, is decreasing in ℓ where the decrease is in some instances strict. As the total probability of being pivotal is the expected value of the conditional probability, where expected values are taken over ℓ, we can conclude that Π has a strictly negative derivative for all p.

If all other individuals play a voting strategy s with voting probability p, then individual i's best response is to vote if $c_i < B(p)$, and not to vote if $c_i > B(p)$. An equilibrium strategy s must thus be a threshold strategy: There is some $\hat{c}$ such that $s(c_i) = 1$ if $c_i < \hat{c}$ and $s(c_i) = 0$ if $c_i > \hat{c}$.

For which values of $\hat{c}$ does the corresponding threshold strategy constitute a symmetric Bayesian equilibrium? For any $\hat{c} \in [\underline{c}, \bar{c}]$ the probability of voting as implied by a threshold strategy with threshold $\hat{c}$ is $F(\hat{c})$. Recall that by assumption F is differentiable, and that $F'(\hat{c}) > 0$ for all $\hat{c} \in (\underline{c}, \bar{c})$. A value $\hat{c}$ is the threshold for an equilibrium threshold strategy if and only if $B(F(\hat{c})) = \hat{c}$ (if $\hat{c} \in (\underline{c}, \bar{c})$) or $B(F(\hat{c})) \geq \hat{c}$ (if $\hat{c} = \underline{c}$) or $B(F(\hat{c})) \leq \hat{c}$ (if $\hat{c} = \bar{c}$). Observe that Remark 1 and the previously noted fact that $F'(\hat{c}) > 0$ imply that $B(F(\hat{c}))$ is differentiable and strictly decreasing in $\hat{c}$. Thus, we are looking in a two dimensional coordinate system with $\hat{c}$ on the horizontal axis and $B(F(\hat{c}))$ on the vertical axis for the intersection of the graph of a differentiable, strictly decreasing function, and the 45° line. Obviously, there can only be one such point of intersection. More precisely, we have the following result, the proof of which is obvious from what has been said so far:

PROPOSITION 1: *Voluntary Majority Voting has a unique symmetric Bayesian equilibrium.*

(i) If $\bar{c} \leq \binom{n-1}{\frac{n-1}{2}}\left(\frac{1}{2}\right)^{n-1}$ *(if n is odd) or* $\bar{c} \leq$

$\binom{n-1}{\frac{n}{2}-1}\left(\frac{1}{2}\right)^{n-1}$ *(if n is even), then the unique equilibrium is that all individuals vote, independent of their participation costs.*

(ii) *If* $\underline{c} \geq 1/2$, *then the unique equilibrium is that no individual ever votes.*

(iii) *Otherwise, there is a unique threshold* c^* $\in (\underline{c}, \bar{c})$ *such that an individual votes if and only if* $c_i \leq c^*$.

The upper boundary for $\bar{c}$ mentioned in part (i) is the value of $B(1)$. The fact that $B(1)$ has the values listed in part (i) follows from calculations in the proof of Remark 1. If $\bar{c}$ is exactly equal to the boundary, then an alternative equilibrium could be constructed in part (i) in which agents do vote if their costs are exactly equal to the boundary value. However, this would be a zero probability event. Recall that we ignore such events. This justifies the claim of uniqueness in Proposition 1 (i). Similar comments apply also to parts (ii) and (iii) of the Proposition.

The Proposition shows that if all conceivable costs of voting are below some boundary, then *Voluntary Majority Voting* is identical to *Compulsory Majority Voting* because everybody will vote voluntarily [part (i)]. If all possible costs of voting are above some boundary, then *Voluntary Majority Voting* is equivalent to *Random Decision Making* because nobody will volunteer to vote [part (ii)]. In intermediate cases, the unique symmetric equilibrium of *Voluntary Majority Voting* implies higher participation costs but better decisions than *Random Decision Making,* and lower participation costs but worse decisions than *Compulsory Majority Voting* [part (iii)].

Our result leaves open whether *Voluntary Majority Voting* has other, not symmetric equilibria. The symmetric equilibrium is arguably the most prominent equilibrium of *Voluntary Majority Voting,* and therefore it seems interesting to explore the properties of this equilibrium. In the following we shall assume without further mentioning that this equilibrium is played.

Although Proposition 1 is simple, we are not aware of any previous paper in which it would have been stated. However, issues related to the ones considered so far in this section have been analyzed before. The probability $\Pi(p)$ that a voter is pivotal has been studied by Nathaniel Beck (1975) and Gary Chamberlain and Michael Rothschild (1981), however, their focus is on the case that n is large. Voting games with endogenous participation similar to the one considered here have been analyzed by Ledyard (1981, 1984). These papers study a model which is more general than ours because it is possible in this model that voters are more likely to prefer one alternative than another. Ledyard focuses on equilibria in which all individuals play the same strategy, and proves existence of such equilibria. A closely related analysis is Palfrey and Rosenthal (1983).

We now turn to the question which of the three procedures discussed so far resolves best the trade-off between quality of decisions and participation costs. The following proposition shows that the answer is unambiguous. This proposition is the main result of this section.

PROPOSITION 2: *Whenever the collective decision induced by Voluntary Majority Voting differs with positive probability from the collective decision induced by Compulsory Majority Voting, then Voluntary Majority Voting Pareto-dominates Compulsory Majority Voting. The same is true for Random Decision Making.*

PROOF:

Step 1: Comparison between *Voluntary Majority Voting* and *Compulsory Majority Voting:*

Suppose that in *Voluntary Majority Voting* all individuals vote if and only if their participation costs are below some common threshold $\hat{c} \in [\underline{c}, \bar{c}]$. As Proposition 1 shows there is only one value of $\hat{c}$ for which this is an equilibrium. However, we can investigate the welfare implications of such behavior independent of whether it is equilibrium behavior or not.

Denote by $U(\hat{c})$ the expected utility of any individual if all individuals adopt the strategy just described. Observe that the expected utility from the unique symmetric equilibrium of *Voluntary Majority Voting* is: $U(c^*)$, and the expected social welfare from *Compulsory Majority Voting* is $U(\bar{c})$. Our task is to prove that $U(c^*) \geq U(\bar{c})$ and $U(c^*) > U(\bar{c})$ if $c^* < \bar{c}$. We shall do so by showing that U is differentiable, and that $U'(\hat{c}) < 0$ if $\hat{c} \in (c^*, \bar{c})$.

We can write $U(\hat{c})$ as follows:

$$U(\hat{c}) = \int_{\underline{c}}^{\hat{c}} \left(\frac{1}{2} + B(F(\hat{c})) - c\right) f(c)\, dc + \int_{\hat{c}}^{\bar{c}} \frac{1}{2} f(c)\, dc.$$

The first integral represents an individual's expected payoff in case that c_i is sufficiently low so that the individual votes. The second integral represents the expected payoff for the case that c_i is so high that the individual does not vote. Conditional on not voting, the expected payoff is ½, since each alternative is equally likely to be chosen. To obtain the expected payoff conditional on voting, we have to add to ½ the benefits from voting, $B(F(\hat{c}))$, but have to subtract the costs of voting, c. Observe that we can rewrite expected utility as follows:

$$U(\hat{c}) = \frac{1}{2} + \int_{\underline{c}}^{\hat{c}} (B(F(\hat{c})) - c) f(c)\, dc.$$

By elementary results of calculus, the function U is differentiable for all $\hat{c} \in (c^*, \bar{c})$ and its derivative is:

$$U'(\hat{c}) = \int_{\underline{c}}^{\hat{c}} B'(F(\hat{c}))F'(\hat{c}) f(c)\, dc + (B(F(\hat{c})) - \hat{c}) f(\hat{c})$$

$$= B'(F(\hat{c}))F'(\hat{c})F(\hat{c}) + (B(F(\hat{c})) - \hat{c}) f(\hat{c}).$$

The first term in this sum is negative because $B'(F(\hat{c})) < 0$, as argued in Remark 1, and, as mentioned before, $F'(\hat{c}) > 0$. Moreover, for $\hat{c} > c^*$ the second term is negative, too. Thus, for $\hat{c} > c^*$, we have: $U'(\hat{c}) < 0$.

Step 2: Comparison between *Voluntary Majority Voting* and *Random Decision Making:*

In the notation of Step 1, we need to show that $U(c^*) \geq$ ½, and that $U(c^*) >$ ½ whenever $c^* > \underline{c}$. Consider the difference: $U(c^*) -$ ½. From the calculations in Step 1 we know:

$$U(c^*) - \frac{1}{2} = \int_{\underline{c}}^{c^*} (B(F(c^*)) - c) f(c)\, dc.$$

If $c^* = \underline{c}$ the right-hand side is evidently zero. Otherwise, $B(F(c)) - c > 0$ for all $c \in (\underline{c}, c^*)$, and the right-hand side is strictly positive.

The comparison between *Voluntary Majority Voting* and *Random Decision Making* is relatively obvious. If under *Voluntary Majority Voting* an individual does not vote, and if the other individuals play a symmetric equilibrium, then from this individual's perspective the probability of each alternative being selected is exactly ½. Thus, it is the same as under *Random Decision Making.* If an individual chooses to vote, it must be that he expects that voting will yield a utility larger than ½. Therefore, under *Voluntary Majority Voting,* if individuals choose to vote voluntarily, each individual's expected utility is larger than under *Random Decision Making.*

To understand the comparison between *Voluntary Majority Voting* and *Compulsory Majority Voting* suppose all individuals vote if and only if their costs are below some threshold $\hat{c}$. Imagine that we raise $\hat{c}$ for all individuals. There will be two effects for the expected utility of some individual i. Firstly, the *direct* effect reflects the change in i's expected utility due to the change in i's own voting behavior. This effect is given by $B(F(\hat{c})) - \hat{c}$, and is positive if $\hat{c}$ is below the equilibrium value c^* and negative otherwise. The second effect is the voting externality. Raising $\hat{c}$ means that all individuals other than i become more likely to vote. For those cost types of individual i which do not vote, this does not matter. From their perspective the probability that each alternative is chosen is independent of the other individual's voting probabilities, and is ½. However, for those types of individual i which do vote, there is a *negative* externality because the probability that individual i is pivotal decreases as the voting probability increases. Thus, for $\hat{c} > c^*$, both effects are negative, and hence their sum is negative. As this is true for all individuals, raising $\hat{c}$ from c^* (*Voluntary Majority Voting*) to $\bar{c}$ (*Compulsory Majority Voting*) makes all individuals worse off, and is thus a Pareto worsening.

III. Conclusion

To conclude, we suggest a reinterpretation of our model as a model of endogenous information acquisition in voting. Specifically, suppose that individuals need to make an effort to find out which of the two alternatives they prefer. Suppose the costs of that effort to individual i are c_i, and assume that an individual i who has not found out which alternative she prefers does not vote. Then the analysis of this paper indicates how different voting rules affect the process of an endogenous information acquisition game.

This question has previously attracted interest in a common value setting (Nicola Persico, 2004). Our contribution is to point out that this issue is also interesting in a private value setting. Even in a private value setting some voting procedures provide better incentives for information acquisition than others. Note that if the participation decision is interpreted as an information acquisition decision, then *compulsory* participation does not seem enforceable. However, our finding that majority voting provides excessive incentives for information acquisition remains valid.

REFERENCES

Beck, Nathaniel. "A Note on the Probability of a Tied Election." *Public Choice*, Fall 1975, *23*(3), pp. 75–79.

Bulkley, George; Myles, Gareth D. and Pearson, Bernhard R. "On the Membership of Decision-Making Committees." *Public Choice*, January 2001, *106*(1), pp. 1–22.

Campbell, Colin. "Large Electorates and Decisive Minorities." *Journal of Political Economy*, December 1999, *107*(6), pp. 1199–217.

Chamberlain, Gary and Rothschild, Michael. "A Note on the Probability of Casting a Decisive Vote." *Journal of Economic Theory*, August 1981, *25*(1), pp. 152–62.

Downs, Anthony. *An economic theory of democracy*. New York: Harper and Row, 1957.

Feddersen, Timothy and Pesendorfer, Wolfgang. "The Swing Voter's Curse." *American Economic Review*, June 1996, *86*(3), pp. 408–24.

Ferejohn, John and Fiorina, Morris. "The Paradox of Not Voting: A Decision Theoretic Analysis." *American Political Science Review*, June 1974, *68*(2), pp. 525–36.

Ghosal, Sayantan and Lockwood, Ben. "Information Aggregation, Costly Voting and Common Values." Mimeo, University of Warwick (first version, 2002; current version, 2003).

Goeree, Jacob and Grosser, Jens. "Costly Voting with Correlated Preferences." Mimeo, CREED, University of Amsterdam, 2003.

Landsberger, Michael and Tsirelson, Boris. "Single Unit Auctions; Still Enigmatic." Mimeo, Haifa University, November 1999.

______. "Correlated Signals Against Monotone Equilibria." Mimeo, Haifa University, April 2000.

Ledyard, John O. "The Paradox of Voting and Candidate Competition: A General Equilibrium Analysis," in George Horwich and James P. Quirk, eds., *Essays in contemporary fields of economics—in honor of Emanuel T. Weiler*. West Lafayette, IN: Purdue University Press, 1981, pp. 54–80.

______. "The Pure Theory of Large Two-Candidate Elections." *Public Choice*, 1984, *44*(1), pp. 7–41.

Osborne, Martin; Rosenthal, Jeffrey and Turner, Matthew. "Meetings with Costly Participation." *American Economic Review*, September 2000, *90*(4), pp. 927–43.

Palfrey, Thomas R. and Rosenthal, Howard. "A Strategic Calculus of Voting." *Public Choice*, 1983, *41*(1), pp. 7–53.

______. "Voter Participation and Strategic Uncertainty." *American Political Science Review*, March 1985, *79*(1), pp. 62–78.

Persico, Nicola. "Committee Design With Endogenous Information." *Review of Economic Studies*, January 2004, *71*(1), pp. 165–91.

Turner, Matthew and Weninger, Quinn. "Meetings With Costly Participation: An Empirical Analysis." Mimeo, University of Toronto, March 2001.