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MENU AUCTIONS, RESOURCE ALLOCATION, AND ECONOMIC INFLUENCE*

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In many examples of competitive bidding (e.g., government construction contracting) the relevant object is either partially divisible or ill-defined, in contrast to much of the recent theoretical work on auctions. In this paper we consider a more general class of auctions, in which bidders name a "menu" of offers for various possible actions (allocations) available to the auctioneer. We focus upon "first-price" menu auctions under the assumption of complete information, and show that, for an attractive refinement of the set of Nash Equilibria, an efficient action always results. Our model also has application to situations of economic influence, in which interested parties independently attempt to influence a decision-maker's action.

I. INTRODUCTION

Economists have recently focused a great deal of attention upon the study of auctions and competitive bidding (see, for example, Milgrom and Weber [1982] and the references contained therein). Nearly all of this literature has concerned the allocation of a single, well-defined, indivisible object.[1] In many circumstances, however, the relevant object may be either ill-defined or

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1. One exception is the literature on share auctions such as Wilson [1979]. We discuss share auctions as an application of our more general model below in Section V.

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divisible. Large government construction projects are, for example, often composed of several distinct component contracts that the government awards simultaneously. During the process of competitive bidding, bidders may submit offers on more than one component and may condition offers upon the set of contracts received (this would be desirable, for example, if they perceive economies or diseconomies of scale or scope). Furthermore, the government typically allows some flexibility in construction specifications. Thus, a contractor may submit several offers with price dependent upon specification.

In this paper we consider a more general class of auctions, in which bidders name a "menu" of offers for the various possible actions available to the auctioneer (e.g., allocations of the components of a construction project). This set of possible actions (allocations) is represented by an abstract choice set over which the bidders and the auctioneer have preferences. Our investigation focuses upon "first-price" menu auctions. By first-price, we mean that bidders pay their announced offers for the allocation ultimately chosen by the auctioneer and that this choice is made to maximize the auctioneer's payoff, given the menus of offers bidders name. Such auctions seem to appear in a variety of contexts.

Throughout, we assume that bidders have complete information (the auctioneer, presumably, is poorly informed). While this assumption is restrictive, it is often a good approximation. In the case of construction contracting, for example, bidders are typically quite well-informed about each others' costs, even though the government is not. Furthermore, although a fully satisfactory theory of menu auctions would certainly allow for incomplete information, we shall soon see that significant complexities arise even when there is no private information.

In first-price complete information auctions of a single indivisible object, the question of allocational efficiency entails no subtlety whatsoever: equilibrium requires that the auctioneer sell the good to the individual who values it most highly.[2] In general, however, the Nash Equilibria of first-price *menu* auctions need not be efficient (we consider an example in Section II). Our central

2. Suppose that this were not the case. Since the equilibrium sales price p^e, cannot exceed the winning bidder's valuation, any individual who valued the object more highly than the winning bidder would do better by naming the price $(p^e + \varepsilon)$ for some $\varepsilon > 0$—an immediate contradiction.

results establish that, for a certain attractive refinement of the Nash Equilibrium set (in which bids correctly reflect relative preferences for the various alternatives), first-price menu auctions always implement efficient actions; furthermore, these "truthful" equilibria possess a strong stability property and are essentially the only equilibria that possess this property. In addition, we characterize the net payoffs that arise in these equilibria.

The model that we develop here can also be applied to a much broader class of problems in which a single individual is endowed with the power to make an important decision, and several affected parties (whose interests conflict) offer rewards or bribes in an attempt to obtain personally desired outcomes. We refer to these situations as instances of "economic influence."

Many examples of economic influence arise when a decision-maker directly allocates resources through rationing. Mundane examples abound. Since waiters ration scarce services among diners, regular diners may endeavor to acquire reputations for tying tips to service quality. Academic secretaries are often responsible to several professors, each of whom may attempt to influence his position in the queue through threats or favors. Parents, concerned with placing their children in a prominent university, may "pull strings" (or offer large contingent donations) to influence the decisions of admissions officials.

More important examples of influence arise in cases of public policy formation and execution. Numerous government officials allocate a variety of contracts and licenses, for which the receipt of kickbacks, though often undocumented, is notorious (especially in many third world countries).[3] More fundamentally, any individual who is affected by government policy has an incentive to influence the policymaker. This influence is sometimes obviously illicit (as with ABSCAM), and at other times more indirect and subtle (as in the case of a government regulator who, upon retiring from government service, acquires a lucrative industry position). In such cases, one may think of the policymaker as auctioning off a complex action choice.

Still another example arises when several parties voluntarily and independently bestow the right to make certain decisions upon a single, common agent, thereby creating a situation of influence. Delegated common agency is particularly prevalent in

3. See, for example, Bhagwati and Desai [1970].

wholesale trade. Numerous products are marketed through merchandise agents and brokers (such as commission merchants and manufacturers' agents), who often represent the potentially conflicting interests of several principals.[4] In fact, the 1972 Census of Wholesale Trade revealed that, of $695 billion in wholesale trade, over $85 billion was transacted through merchandise agents and brokers ($19 billion through commission merchants and $23 billion through manufacturers' agents). Similar institutions are also observed in a number of retail industries including travel, insurance, and real estate. While our model does not describe the process and strategy of delegation, it does apply to post-delegation behavior. One interesting implication of our model is its suggestion that common agency may serve to facilitate collusion among competitors (that is, a noncooperative choice of incentive schemes can lead to a cooperative marketing outcome, for example).

The paper is organized as follows. Section II motivates the investigation with a simple example of a menu auction. Section III describes the model. Section IV presents our results. Finally, Section V closes the paper by discussing the implications of these results for some of the problems mentioned above.

II. The Problem

Consider the sale of an object (X), which can be divided into two pieces, labeled X_1 and X_2. Suppose that there are two bidders, A and B, who make nonnegative offers for the object or parts thereof. In particular, each bidder has the flexibility to name a price for the entire object, X_1 alone, X_2 alone, and nothing. Suppose that the auctioneer is indifferent with respect to the final allocation, and that bidders' gross payoffs (measured in dollars) are as follows:

	Value to A	Value to B
X_1	6	5
X_2	5	6
X	8	7
Nothing	0	0

4. While many manufacturers' agents refuse to serve two principals with directly competing products, product lines may be quite similar, or perhaps complementary. Even when substitutability between principals' products is low, conflict still arises over the allocation of the agent's marketing efforts.

TABLE I

Equilibrium	(a)	(b)	(c)	(d)	(e)	(f)
A's offer for:						
nothing	0	0	0	0	0	0
X_1	0	0	2	0	5	1
X_2	0	3	3	2	0	0
X	7	6	6	5	7	3
B's offer for:						
nothing	0	0	0	0	0	0
X_1	0	3	3	3	0	1
X_2	0	0	2	0	2	2
X	7	6	6	5	7	3
Equilibrium allocation:						
A	X	X_2	X_2	X_2	X_1	X_1
B	0	X_1	X_1	X_1	X_2	X_2
Net payoffs:						
A	1	2	2	3	1	5
B	0	2	2	2	4	4
Auctioneer	7	6	6	5	7	3

(notice that both A and B evidently regard X_1, and X_2 as substitutes). Clearly, the efficient allocation is to assign X_1 to A, and X_2 to B.

What allocations can arise in Nash Equilibrium? Table I exhibits six equilibria for this game, labeled (a) through (f) (there are others, as well).[5] Note, in particular, that a variety of allocations are possible.

How are suboptimal allocations sustained in equilibrium? Table I presents four examples of inefficiency: equilibria (a) through (d). In (a) each bids on X, but neither bids separately on the two component projects (given that the other fails to do so, this is optimal). In (b) and (d) each bids on X, and in addition A bids on X_2, while B bids only on X_1. A's failure to bid on X_1 is justified by B's failure to bid on X_2, and vice versa. In equilibrium (c) A and B bid on the entire object and both of its components sepa-

5. In equilibrium (a), for example, bidder A's net payoff of 1 is the best he can do given B's strategy—he must offer at least 7 in order to get either X, X_1, or X_2 and of these X offers him the highest payoff (and a net payoff that is better than his receiving nothing). The auctioneer, on the other hand, is indifferent between giving X to A or B, and resolves his indifference in favor of A. (It is, in fact, a general property of equilibrium that the auctioneer always resolves his indifference in favor of the allocation with the highest social payoff. This problem, of course, also occurs in complete information auctions of a single indivisible object and, as there, is an artifact of the infinite divisibility of money in the model.)

rately. However, A's bid on X_1 is not large enough to induce a "serious" bid from B on X_2, and B's bid on X_2 is not large enough to induce a "serious" bid from A on X_1.

Thus, it appears that inefficient allocations arise from the failure of bidders to make "serious" offers on every alternative. This failure is sustained by the norm of making "serious" offers only on particular alternatives. If, for instance, each bidder feels that his competitors will treat a project as indivisible, he will do so as well, even if potential divisibilities are known to exist.

One might speculate that the introduction of uncertainty would eliminate these inefficient equilibria. For example, if each player expected the other to make serious offers on all alternatives with some small probability, it might be optimal for him to do so as well. This notion is difficult to formalize, however. In particular, trembling hand perfectness [Selten, 1975] does *not* eliminate the inferior equilibria. Similarly, the same difficulty would persist even if the bidders had incomplete information about each others' preferences. So long as the Bayes strategy of one player never entails making a serious offer on some alternative, it may be optimal for the other player to select a strategy with the same property.

It may, of course, be possible to design an alternative auction mechanism that guarantees an efficient allocation. Our objective is, instead, to pursue a more detailed investigation of the properties of allocations arising from first-bid menu auctions, since such mechanisms appear in a variety of contexts.

III. The Model

Consider a game in which an "auctioneer" selects an action affecting the well-being of M "bidders," each of whom offers a menu of payments contingent on the action chosen. We denote the set of bidders by $\mathcal{J} = \{i\}_{i=1}^{M}$, and subsets by $J \subseteq \mathcal{J}$. $\bar{J}$ will indicate the complement of J. However, in the specific case where $J = \{i\}$, we shall typically write "$-i$" instead of $\bar{J}$. Possible choices for the auctioneer are given by a finite set S.[6] Bidder i receives gross monetary payoffs described by the function g_i: $S \rightarrow R$, while the function d: $S \rightarrow R$ indicates the disutility (in monetary terms) that the auctioneer experiences in taking each possible

6. This assumption is made for ease of exposition only. All of our theorems hold when S is a compact set and the payoff functions $\{g_i\}_{i=1}^{M}$ and d (defined below) are continuous.

action. In the case of construction contracting, for example, $g_i(\cdot)$ would represent bidder i's costs of completing various portions and specifications of a project (a nonpositive number in this case) while $d(\cdot)$ reflects the auctioneer's preferences over project specification (the allocation of contracts among bidders is, aside from price, presumably a matter of indifference to the auctioneer). For all $s \in S$, $J \subseteq \mathcal{J}$, let $G_J(s) \equiv \Sigma_{i \in J}\, g_i(s)$. For all $J \in \mathcal{J}$, let

$$S^J \equiv \underset{s \in S}{\operatorname{argmax}}\ [G_J(s) - d(s)],$$

where S^J contains actions that yield the highest joint payoff to the auctioneer and members of group J. Define $S^* \equiv S^{\mathcal{J}}$; S^* contains efficient actions.

In the extensive form of this game the M bidders simultaneously offer contingent payments (negative in the case of contract bidding) to the auctioneer, who subsequently chooses an action that maximizes his total payoff. The strategy of each bidder consists of a function f_i: $S \to R$; that is, he offers the auctioneer a monetary reward of $f_i(s)$ for selecting action s. The set of feasible strategies for each bidder is given by

$$\mathcal{F}_i = \{f_i \mid f_i(s) \geqq k_i(s) \text{ for all } s \in S\}.$$

The function $k_i(\cdot)$ places lower bounds on the bids offered for each action. These lower bounds reflect the limited ability of bidders to extract payments from the auctioneer. We specify $k_i(\cdot)$ quite generally in order to allow application of our model to the various problems mentioned above. For most applications certain bounds are quite natural. In particular, for competitive bidding, a bidder certainly cannot demand payment (represented by a negative value of f) if the auctioneer fails to award him any portion of a project.

For a particular strategy f_i, bidder i's net payoff at each action s is given by the function $n_i(s) \equiv g_i(s) - f_i(s)$. Following earlier convention, we define $F_J(s) \equiv \Sigma_{i \in J}\, f_i(s)$ and $N_J(s) \equiv \Sigma_{i \in J}\, n_i(s)$ for all $s \in S$, $J \subseteq \mathcal{J}$ (thus, $N_J(s) = G_J(s) - F_J(s)$).

In a first-price menu auction, the auctioneer chooses an action that maximizes his total payoff—i.e., given an element of $\Pi_{i=1}^{M} \mathcal{F}_i$, the auctioneer selects an element of the set,

$$I^*(\{f_i\}_{i=1}^{M}) \equiv \underset{s \in S}{\operatorname{argmax}}\ [F_{\mathcal{J}}(s) - d(s)].$$

Since a menu auction Γ is completely specified once the action space, reward spaces, disutilities, and gross payoffs have been specified, we may write $\Gamma = [S, \{k_i\}_{i=1}^M, \{g_i\}_{i=1}^M, d]$. $(\{f_i^0\}_{i=1}^M, s^0)$ is a *Nash Equilibrium* for the auction Γ if $f_i^0 \in \mathcal{F}_i$ for all i, $s^0 \in I^*(\{f_i^0\}_{i=1}^M)$, and—given $\{f_j^0\}_{j \neq i}$—no bidder i has a feasible strategy that would yield him a net payoff greater than $n_i(s^0)$.

Because it is notationally complex to carry around the $k_i(s)$'s in all of our proofs, we establish here that any menu auction can be transformed into an equivalent auction where every bid is bounded below by zero. In particular, consider an auction $\Gamma = [S, \{k_i\}_{i=1}^M, \{g_i\}_{i=1}^M, d]$. Define a new auction $\tilde{\Gamma} = [S, \{\tilde{k}_i\}_{i=1}^M, \{\tilde{g}_i\}_{i=1}^M \tilde{d}]$, where

$$\tilde{k}_i(\mathrm{s}) \equiv 0 \qquad \text{for all } i \in \mathcal{J},\ s \in S;$$

$$\tilde{g}_i(s) \equiv g_i(s) - k_i(s) \qquad \text{for all } i \in \mathcal{J},\ s \in S;$$

$$\tilde{d}(s) \equiv d(s) - \sum_{i=1}^{M} k_i(s) \qquad \text{for all } s \in S.$$

The following result establishes that these two games are strategically equivalent.

LEMMA 1. Consider strategies $\{f_i\}_{i=1}^M$ and $\{\tilde{f}_i\}_{i=1}^M$, where for all $s \in S$ and $i \in \mathcal{J}$,

$$\tilde{f}_i(s) = f_i(s) - k_i(s).$$

Then, $\tilde{f}_i$ is a feasible strategy for bidder i in auction $\tilde{\Gamma}$ if and only if f_i is a feasible strategy for bidder i in game Γ. Furthermore, the net payoffs resulting from strategies $\{\tilde{f}_i\}_{i=1}^M$ in auction $\tilde{\Gamma}$ are identical to those resulting from strategies $\{f_i\}_{i=1}^M$ in auction Γ.

Proof of Lemma 1. Clearly $\tilde{f}_i(s) \geqq 0$ if and only if $f_i(s) \geqq k_i(s)$ so that $\tilde{f}_i$ is feasible in $\tilde{\Gamma}$ if and only if f_i is feasible in Γ. To verify our claim regarding net payoffs, first note that

$$s^0 \in \operatorname*{argmax}_{s \in S} \sum_{i=1}^{M} f_i(s) - d(s)$$

if and only if

$$s^0 \in \operatorname*{argmax}_{s \in S} \sum_{i=1}^{M} \tilde{f}_i(s) - \tilde{d}(s)$$

so that the same action results from $\{\tilde{f}_i\}_{i=1}^M$ in auction $\tilde{\Gamma}$ as results from $\{f_i\}_{i=1}^M$ in auction Γ. But $\tilde{g}_i(s^0) - \tilde{f}_i(s^0) = g_i(s^0) - f_i(s^0)$ for every bidder i, so the net payoffs received by every bidder are identical (the net payoff of the auctioneer is identical as well).

Q.E.D.

Strategic equivalence in this sense is easily seen to imply that $(\{f_i\}_{i=1}^M, s^0)$ is a Nash Equilibrium of auction Γ if and only if $(\{\tilde{f}_i\}_{i=1}^M, s^0)$ is a Nash Equilibrium of auction $\tilde{\Gamma}$. Furthermore, a similiar correspondence results for the refinement of the Nash Equilibrium set that we shall introduce below. Given this fact, we shall henceforth assume without loss of generality that $k_i(s) \equiv 0$ for all $i \in \mathcal{J}$ and $s \in S$. We define

$$\mathcal{F} \equiv \{f \mid f(s) \geqq 0 \qquad \text{for all } s \in S\}.$$

The model of first-price menu auctions that we have described here bears a strong formal resemblance to the problem of choosing between several mutually exclusive public goods. In particular, we can think of the auctioneer's actions as public projects; every player expresses a willingness to contribute to each project, and the project with the largest total support is implemented. Given this analogy, one might expect inefficiency necessarily to arise from a tendency for each player to "free ride" on the contributions offered by others with similar interests. As we demonstrate below, however, this is not the case.

Nevertheless, the public goods analogy does prove useful. A large branch of the literature on public goods concerns the design of mechanisms that implement efficient social choices. While the objective of this literature differs from ours (we are concerned with description, rather than design) and while the game described above does not correspond to any mechanism discussed in the literature, it is illuminating to explore parallels.

Consider, in particular, the Groves-Clarke mechanism.[7] In this context, players would announce payoff functions $\tilde{g}_i$. Note that this is completely equivalent to announcing strategies in $\mathcal{F}$, up to a scalar. As in our game, the Groves-Clarke mechanism selects the project with the largest announced net payoff (reward). There are, however, some fundamental differences arising from the methods of calculating equilibrium payments. In our model,

7. See Clarke [1971] and Groves [1973]. Our discussion specializes to the case of complete observability.

players pay the amount of their announcements $f_i(s^0)$. In the Groves-Clarke mechanism a player i pays the difference between the announcements of other players at the action that would be selected in his absence $\tilde{G}_{-i}(s^{-i})$, where $s^{-i} \in I^*[\{\tilde{g}_j\}_{j \neq i}]$, and their announcements at the equilibrium action, $\tilde{G}_{-i}(s^0)$. Thus, a change in a player's announcement only affects his payoff insofar as it alters the public decision. Adding a constant to his announcement at all s, for example, has no effects (in fact, allowable announcements in the Groves-Clarke scheme entail a normalization). In contrast, changes in absolute levels do alter payoffs in first-price menu auctions. It is essentially for this reason that bidders in first-price menu auctions do not have dominant strategies, as do participants in the Groves-Clarke allocation scheme. Nevertheless, as we shall see, reference to the Groves-Clarke mechanism provides some useful guidance in our analysis of equilibria.

IV. Results

We begin our analysis by completely characterizing the set of Nash Equilibria for first-price menu auctions.

LEMMA 2. Consider a first-price menu auction Γ. $(\{f_i\}_{i=1}^M, s^0)$ is a Nash Equilibrium if and only if

(i) $f_i \in \mathcal{F}$ for all $i \in \mathcal{J}$

(ii) $s^0 \in I^*(\{f_i\}_{i=1}^M)$

(iii) $[g_i(s^0) - d(s^0)] - [g_i(s) - d(s)] \geqq [F_{-i}(s - F_{-i}(s^0))]$ for all $i \in \mathcal{J}$, $s \in S$

(iv) there exists $s_i \in I^*(\{f_i\}_{i=1}^M)$ such that $f_i(s_i) = 0$ for all $i \in \mathcal{J}$.

Proof of Lemma 2. (a) *Necessity.* Condition (i) simply states that strategies are feasible. (ii) follows from payoff maximization on the part of the auctioneer. For condition (iii) note that in any Nash Equilibrium where the auctioneer chooses s^0, it must be the case that

$$g_i(s) - [F_{\mathcal{J}}(s^0) - d(s^0)] - \{[F_{-i}(s) - d(s)]\} \leqq g_i(s^0) - f_i(s^0)$$

for all $s \in S$ and $i \in \mathcal{J}$. Otherwise, bidder i could do strictly better by playing $\tilde{f}$, where $\tilde{f}$ differs from f_i only at $\tilde{s}$ (an action for which the above condition is violated), and $\tilde{f}_i(\tilde{s}) = [F_{\mathcal{J}}(s^0) - d(s^0)] -$

$[F_{-i}(s) - d(s)] + \varepsilon$ four $\varepsilon > 0$ sufficiently small (this is feasible since s^0 satisfies condition (ii)). Condition (iii) follows immediately by rearranging this inequality. For condition (iv), note that if there did not exist such an s_i, then bidder i could lower $f_i(s)$ for all $s \in I^*(\{f_i\}_{i=1}^M)$ without changing the auctioneer's choice. This would, of necessity, leave him strictly better off.

(b) *Sufficiency*. Suppose that $(\{f_i\}_{i=1}^M, s^0)$ is *not* a Nash Equilibrium. Then, without loss of generality, bidder i may play a feasible alternative strategy $\tilde{f}_i$ that leaves him strictly better off when the auctioneer chooses some $\tilde{s} \in I^*(\tilde{f}_i, \{f_j\}_{j \neq i})$. Thus,

$$(1) \quad \tilde{f}_i(\tilde{s}) + F_{-i}(\tilde{s}) - d(\tilde{s}) \geqq \tilde{f}_i(s) + F_{-i}(s) - d(s) \text{ for all } s \in S$$

and

$$(2) \qquad g_i(\tilde{s}) - \tilde{f}_i(\tilde{s}) > g_i(s^0) - f_i(s^0).$$

Substituting for $[g_i(\tilde{s}) - g_i(s^0)]$ in (2) from condition (iii) implies that

$$\tilde{f}_i(\tilde{s}) < f_i(s^0) + [F_{-i}(s^0) - d(s^0)] - [F_{-i}(\tilde{s}) - d(\tilde{s})]$$

or, collecting terms, that

$$\tilde{f}_i(\tilde{s}) < [F_{\mathscr{I}}(s^0) - d(s^0)] - [F_{-i}(\tilde{s}) - d(\tilde{s})].$$

Using (iv), we have

$$\tilde{f}_i(\tilde{s}) < [0 + F_{-i}(s_i) - d(s_i)] - [F_{-i}(\tilde{s}) - d(\tilde{s})].$$

Since $\tilde{f}_i$ is feasible, condition (i) implies that

$$\tilde{f}_i(\tilde{s}) < [\tilde{f}_i(s_i) + F_{-i}(s_i) - d(s_i)] - [F_{-i}(\tilde{s}) - d(\tilde{s})]$$

or

$$[\tilde{f}_i(\tilde{s}) + F_{-i}(\tilde{s}) - d(\tilde{s})] < [\tilde{f}_i(s_i) + F_{-i}(s_i) - d(s_i)].$$

But this contradicts (1) for $s = s_i$.

Q.E.D.

Our simple example in Section II suggests that a large number of contingent offers will typically satisfy the four conditions of Lemma 2. Are all of these equally plausible? We argue that they are not. The argument proceeds in two steps. First, we analyze a subclass of equilibria with certain appealing characteristics. These equilibria may be "focal," particularly in situations in which no communication occurs between bidders. This subset of equilibria is always nonempty and always results in an efficient action choice. In addition, we precisely characterize the net payoffs obtainable in this class of equilibria. For the simplest menu auctions (two bidders, no inherent auctioneer preferences over his action set), these payoffs are uniquely determined. Second, we demonstrate that for situations in which nonbinding communication is possible between bidders, these equilibria have a strong stability property, and are essentially the *only* equilibria possessing this property.

As suggested in Section III, our point of departure is the Groves-Clarke mechanism. Recall that for this mechanism, each player has a dominant strategy: reveal the truth. In essence, his announced willingness-to-pay the planner for changing from one project to another is exactly equal to the difference between his gross payoffs from these two projects. Similarly, in menu auctions, we might envision a bidder offering rewards that mirror the relative values which he attaches to the various actions. This motivates the following definition:[8]

DEFINITION. $f_i(\cdot)$ is said to be a *truthful strategy relative to* s^0 if and only if for all $s \in S$, either

$$\text{(i) } n_i(s) = n_i(s^0),$$

or,

$$\text{(ii) } n_i(s) < n_i(s^0), \qquad \text{and } f_i(s) = 0.$$

$(\{f_i\}_{i=1}^M, s^0)$ is said to be a *Truthful Nash Equilibrium* if and only if it is a Nash Equilibrium and $\{f_i\}_{i=1}^M$ are truthful strategies relative to s^0.

Note that in any truthful equilibrium each bidder offers a reward for action s that exactly reflects his "net willingness-to-pay" for s as opposed to s^0 (for some actions, he may offer too much compensation due to the nonnegativity constraint in $\mathcal{F}$). Truthful strategies are extremely simple, and truthful equilibria (if they exist) may be quite "focal."

8. Here we define Truthful Nash Equilibrium only for the case where $k_i(s) = 0$ for all i and $s \in S$. More generally, condition (ii) of the definition would be replaced by "(ii) $n_i(s) < n_i(s^0)$, and $f_i(s) = k_i(s)$." Note that the correspondence discussed above in relation to Lemma 1 holds for Truthful Nash Equilibria—that is, $(\{f_i\}_{i=1}^M, s^0)$ is a Truthful Nash Equilibrium in the normalized game $\tilde{\Gamma}$ if and only *if* $(\{f_i\}_{i=1}^M, s^0)$ is a Truthful Nash Equilibrium in the original game Γ.

In addition, as we now show, a bidder can essentially restrict himself to using truthful strategies without loss: every best-response set contains a truthful strategy.

THEOREM 1. Consider a first-price menu auction Γ and any bidder i. For any set of offers by his opponents, $\{f_j\}_{j \neq i}$, bidder i's best-response correspondence contains a truthful strategy.

Proof of Theorem 1. Consider any element of i's best-response correspondence f_i, and suppose that if f_i is used, the auctioneer selects action s^0. Suppose that f_i is not a truthful strategy. Define the strategy $\tilde{f}_i$ such that $\tilde{f}_i(s^0) = f_i(s^0)$ and $\tilde{f}_i$ is a truthful strategy relative to s^0. Using this strategy must yield the same net payoff to i as does using f_i. To see this, note first that if s^0 is still chosen under $\tilde{f}_i$, then i receives the same net payoff as under f_i. If s^0 is not chosen, then it must be that $\tilde{f}_i(\tilde{s}) > f_i(\tilde{s})$, where $\tilde{s}$ is the auctioneer's preferred choice under $\tilde{f}_i$. But, since $\tilde{f}_i$ is a truthful strategy relative to s^0, this implies that

$$g_i(\tilde{s}) - \tilde{f}_i(\tilde{s}) = g_i(s^0) - \tilde{f}_i(s^0) = g_i(s^0) - f_i(s^0).$$

Q.E.D.

Thus, the set of Truthful Nash Equilibria is an appealing refinement of the Nash set. Our first task is to establish the existence of these equilibria, and to explore their properties. Later we shall show that whenever (implicit or explicit) communication among the bidders is possible, truthful equilibria also possess a strong stability property that strengthens the case for using this refinement.

In what follows, it will be convenient to refer to the following sets of net payoff vectors. Let

$$\Pi_\Gamma(s) \equiv \{n \in R^M \mid \text{for all } J \subseteq \mathcal{J}, \quad N_J \leqq [G_{\mathcal{J}}(s) - d(s)] - [G_{\bar{J}}(s^{\bar{J}}) - d(s^{\bar{J}})]\},$$

where $s^{\bar{J}} \in S^{\bar{J}}$, $N_J = \Sigma_{i \in J} n_i$, and by convention $G_{\mathcal{J}}(s^{\bar{\mathcal{J}}}) - d(s^{\bar{\mathcal{J}}}) \equiv \min_{s \in S} d(s)$. We also define the Pareto efficient frontier of $\Pi_\Gamma(s)$:[9]

$$E_\Gamma(s) \equiv \{n \in R^M \mid n \in \Pi_\Gamma(s) \text{ and there does not exist } n' \in \Pi_\Gamma(s), \text{ with } n' \geqq n\}.$$

9. We adopt the convention that for two vectors x and y, $x \geqq y$ means that $x_i \geqq y_i$ for all i and $x \neq y$.

Clearly the set $E_\Gamma(s)$ is nonempty. Also note that for $s', s'' \in S^*$, $\Pi_\Gamma(s') = \Pi_\Gamma(s'') \equiv \Pi_\Gamma(S^*)$ and $E_\Gamma(s') = E_\Gamma(s'') \equiv E_\Gamma(S^*)$.

We are now in a position to state our fundamental result concerning the set of Truthful Nash Equilibria.

THEOREM 2. Consider a first-price menu auction Γ. In all Truthful Nash Equilibria the auctioneer selects $s^0 \in S^*$, and the bidders receive payoffs in $E_\Gamma(S^*)$. Furthermore, any net payoff vector $n \in E_\Gamma(S^*)$ can be supported by a Truthful Nash Equilibrium.

Proof of Theorem 2. The proof can be found in the Appendix.

It should not be too surprising that truthful equilibria involve an efficient action choice—the auctioneer internalizes variations in aggregate payoffs. However, given the many discontinuities and nonconvexities apparent in the structure of our model, existence is somewhat more surprising.[10] In addition, Theorem 2 provides a nice characterization of the set of net payoffs that bidders can receive in a truthful equilibrium: these payoffs must lie in the set $E_\Gamma(S^*)$. In fact, for first-price menu auctions in which there are only two bidders and the auctioneer has no inherent preferences over his decision set, Theorem 2 leads to the following result.

COROLLARY 1. Consider a first-price menu auction Γ, in which there are two bidders and the auctioneer has no inherent preferences over his action set. There exist unique Truthful Nash Equilibrium bids (s^0 is also unique if S^* is a singleton), and in a truthful equilibrium bidder i receives a net payoff of $G_{\not{\$}}(S^*) - g_j(s^j)$ (where $j \neq i$ and $s^j \varepsilon S^j$).

Proof of Corollary 1. In this special case $\Pi_\Gamma(S^*)$ involves three constraints:

$$n_1 \leqq G_{\not{\$}}(S^*) - g_2(s^2) \tag{3}$$

$$n_2 \leqq G_{\not{\$}}(S^*) - g_1(s^1) \tag{4}$$

$$n_1 + n_2 \leqq G_{\not{\$}}(S^*). \tag{5}$$

10. It is also interesting to note that Truthful Nash Equilibrium strategies correspond to a set of nonlinear Lindahl prices—given their strategies, the interested parties all prefer that the auctioneer pick allocation s^*. For a more general discussion of Lindahl equilibria in public good environments devoid of linear structure, see Mas-Colell [1980].

However, by the definitions of s^1 and s^2, it is clear that (5) is redundant. The result then follows from the definition of $E_\Gamma(S^*)$ and Theorem 2.

Q.E.D.

Corollary 1 allows us to note an interesting aspect of the Truthful Nash Equilibrium net payoffs for these simple menu auctions: they are identical to the Groves-Clarke mechanism equilibrium net payoffs (ignoring lump sum transfers). The reason for this is fairly straightforward. As we noted above, in the Groves-Clarke mechanism bidder 1 would pay $[g_2(s^2) - g_2(s^*)]$ (where $s^* \in S^*$). Here, in a Nash Equilibrium that implements s^*, bidder 1 must have no cheaper way to ensure that s^* is chosen than by using his equilibrium strategy. But, when bidder 2's offer is truthful, this involves an offer for s^* by bidder 1 of exactly $[g_2(s^2) - g_2(s^*)]$ (since he will set $f_1(s^2) = 0$).

We have argued above that truthful equilibria are appealing, perhaps even focal. For environments in which bidders can communicate with each other, a strong additional justification for focusing our attention exclusively upon truthful equilibria is available.

One natural question to ask is whether, in a particular equilibrium, any coalition of bidders has an incentive to communicate among themselves, with the intention of arranging a stable, mutually preferable joint deviation. We wish to restrict attention to the set of equilibria for which no such *coalitional* deviation is possible. This stability requirement corresponds to the notion of a Coalition-Proof Nash Equilibrium, as introduced in Bernheim and Whinston [1984]. (Peleg [1984] has independently developed the same notion, which he labels "quasi-coalitional equilibria".) As this may be an unfamiliar concept, we provide a brief exposition here and refer the reader to these other papers for further discussion.

Formally, we define coalition-proofness as follows. First, for any subgroup J and strategies $\{f_i\}_{i \in \bar{J}}$, we define the *subgroup J component game relative to* $\{f_i\}_{i \in \bar{J}}$ as follows:

$$\Gamma/\{f_i\}_{i \in \bar{J}} = (S, \{g_i\}_{i \in J}, \{k_i\}_{i \in J}, d - \sum_{i \in J} f_i).$$

That is, $\Gamma/\{f_i\}_{i \in \bar{J}}$ is the restriction of the game Γ to bidders in subgroup J, where the strategies of the bidders in $\bar{J}$ are held fixed.

Note that $\{f_i\}_{i \in \bar{J}}$ causes the auctioneer to act as though he has preferences over S.

DEFINITION. (i) In a first-price menu auction Γ with a single bidder ($M = 1$), (f_1^0, s^0) is a *Coalition-Proof Nash Equilibrium* if and only if it is a Nash equilibrium.

(ii)-(a) For a first-price menu auction Γ, where $M > 1$, $(\{f_i^0\}_{i=1}^M, s^0)$ is *self-enforcing* if for all $J \subset \mathcal{J}$ (where $J \neq \mathcal{J}$), $(\{f_i^0\}_{i \in J}, s^0)$ is a Coalition-Proof Nash Equilibrium in the subgroup J component game $\Gamma / \{f_i^0\}_{i \in \bar{J}}$.

(ii)-(b) $(\{f_i^0\}_{i=1}^M, s^0)$ is a *Coalition-Proof Nash Equilibrium* if it is a self-enforcing Nash Equilibrium and offers net payoffs to the M bidders that are not Pareto dominated by any other self-enforcing Nash Equilibrium.

Note that this definition is recursive. For one player, any maximizing choice is obviously coalition-proof. For two players, no Nash Equilibrium can be upset by single player coalition, but only Pareto-undominated Nash Equilibria cannot be disturbed by two-player coalitions.[11] For $M = 3$, a Nash equilibrium is coalition-proof if each pair of players achieves a Pareto-undominated equilibrium in the game formed by taking the action of the third player as given, and if the resulting payoffs in the three-player game are not Pareto dominated by any other equilibrium satisfying this condition. For $M > 3$ players, we require that for all groups of size less than M, no coalition-proof equilibrium (sustainable agreement) exists in the component game for that coalition which makes all of its members better off. Pareto-undominated members of this set of M-player equilibria are designated coalition-proof (Pareto-dominated members can be upset by the entire set of players).

11. Note that in considering joint deviations by coalitions of bidders, we have implicitly allowed them to resolve the auctioneer's indifference within the set I^*. While a recalcitrant auctioneer might refuse to resolve his indifference in the desired way, the bidders could eliminate this indifference by simply adding an infinitesimal payoff to the desired allocation. This would, of course, no longer be an equilibrium (for the two-bidder case; self-enforcing, more generally), since the addition can be arbitrarily small. Once again, however, this problem is an artifact of the infinite divisibility of money in the model and disappears if money has some smallest unit (however small). This assumption also implies that the auctioneer is a completely passive follower and need not be considered in searching for mutually beneficial coalitional deviations.

We can now prove a striking result: all Truthful Nash Equilibria are coalition-proof, and furthermore, the set of net payoffs for the bidders that can arise in Coalition-Proof Nash Equilibria *exactly coincides* with those arising in Truthful Nash Equilibria. Thus, truthful equilibria not only possess an inherent appeal, but are also (essentially) the *only* Nash Equilibria that are stable when (nonbinding) communication is possible.[12]

THEOREM 3. Consider a first-price menu auction Γ. In all Coalition-Proof Nash Equilibria the auctioneer selects $s^0 \in S^*$, and the bidders receive payoffs in $E_\Gamma(S^*)$. Furthermore, all Truthful Nash Equilibria are coalition-proof. Thus, any payoff vector $n \in E_\Gamma(S^*)$ can be supported by a Coalition-Proof Nash Equilibrium.

Proof of Theorem 3. The proof can be found in the Appendix.

Note that any equilibrium which is not truthful, but which yields truthful equilibrium payoffs, differs from a truthful equilibrium in an irrelevant way. Returning to our earlier example, column (f) in Table I is the unique Truthful Nash Equilibrium (recall that for the case of two bidders, and no auctioneer preferences, $E_\Gamma(S^*)$ is a singleton). One may, however, vary bids for actions outside of I^* without affecting the equilibrium (e.g., A could bid 1 for X_2). While truthful equilibria are therefore not literally the only stable equilibria, truthful outcomes are the only stable outcomes.

Finally, note that Theorems 2 and 3, through their use of the set $E_\Gamma(S^*)$, provide a remarkably simple way to calculate equilibrium net payoffs in menu auctions. Consider, for example, the following menu auction with three bidders, three possible actions, and no auctioneer preferences over these choices.

	$g_1(\cdot)$	$g_2(\cdot)$	$g_3(\cdot)$
s_1	10	10	10
s_2	16	13	0
s_3	0	0	13

12. By "essentially" we mean that the only stable (coalition-proof) equilibria that are not truthful differ from Truthful Nash Equilibria in an irrelevant way off the equilibrium path.

For this game, the set of Truthful (and Coalition-Proof) Nash Equilibrium net payoffs is given by (observe that several of the constraints in $\Pi_\Gamma(S^*)$ turn out to be redundant):

$$\begin{aligned} n_1 + n_2 &\leqq 17 \\ n_1 &\leqq 10 \\ n_2 &\leqq 10 \\ n_3 &= 1. \end{aligned}$$

In addition, we can of course also easily construct the set of Truthful Nash Equilibrium strategies.[13]

V. Applications

We have already described the application our menu auction model to situations of contract bidding. In this section we briefly discuss the implications of our results for some of the other problem areas outlined in the introduction.

A. Share Auctions

One special case of our model occurs when ownership shares for an item are auctioned off. Such "share auctions" have, for example, been proposed in the past as an alternative means of auctioning leases on Outer Continental Shelf tracts (see Wilson [1979]). For simplicity, suppose that there are two bidders, 1 and 2. Then, a share auction can be represented by setting

$$S \equiv \{(x_1,x_2) \in R^2 \mid x_1 + x_2 = 1, \quad x_1 \geqq 0, x_2 \geqq 0\}$$
$$k_i(s) \equiv 0 \quad \text{for all } s \in S, \quad i = 1,2$$
$$d(s) \equiv 0 \quad \text{for all } s \in S.$$

It remains, of course, to specify the gross payoff functions (g_1,g_2). Suppose, first, that the value of the entire item is known with certainty to be V (or, alternatively, that both bidders are risk-neutral, have no proprietary information, and attach an expected value to the item of V). Then $g_i(s) \equiv V \cdot x_i$ for $i = 1,2$; that is, the auction is one of pure division. For this auction $S^* = S$,

13. It is interesting to note that, in this example, the set of Pareto-undominated Nash Equilibria does not correspond to $E_\Gamma(S^*)$. The payoffs $(n_1, n_2, n_3) = (16,0,0)$, for example, can be supported in a Nash Equilibrium that implements s_2. However, as Theorem 3 indicates, the strategy profile that does this is vulnerable to coalitional deviations, making it an unlikely outcome in situations with possibilities for communication.

and our results imply that the auctioneer's payoff, which is generally given by $[g_1(s^1) + g_2(s^2) - G_{\mathcal{I}}(S^*)]$ is exactly equal to V (since $g_1(s^1) = g_2(s^2) = G_{\mathcal{I}}(S^*)$). Thus, the auctioneer gets exactly the same revenue in the share auction as he would by selling the item in a "unit" auction (where $S \equiv \{(1,0),(0,1)\}$).

Interestingly, this conclusion is at odds with that arrived at by Wilson [1979], who argues that an auctioneer can do much worse (in terms of revenue) in a share auction of an item of certain value than he does in a unit auction of that item. The reason lies in a slight difference between Wilson's share auction mechanism and ours. In Wilson's share auction model, bidders name demand schedules as a function of the price per share $x_i(p)$, and the auctioneer picks a price to "clear the market"; i.e., such that $x_1(p) + x_2(p) = 1$. This prevents certain natural bidding strategies. For example, if bidder 2 offers the bids depicted in Figure I, bidder 1 can only acquire the entire item by paying V. He cannot get it by offering say $V - \varepsilon$ (as he can in our model) because that price will not then clear the market. In fact, his best response is then to offer the same set of offers as bidder 2 so that the auctioneer's revenue is $V/2$ (this is actually Wilson's equilibrium).

Wilson then demonstrates by example that the same result can hold when the value of the item is uncertain, and bidders are risk-averse. Let us now examine Wilson's example using our model. The value of the item is stochastically distributed, $\tilde{V} \sim N(m,\sigma^2)$. Bidder i has an expected utility function over net payoffs of the

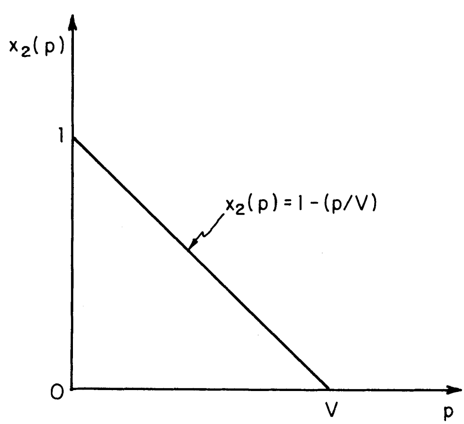

FIGURE I

form $u(n) = -e^{-r_i n}$. Then we can write the (certainty equivalent) gross payoff to bidder i from x_i shares as[14]

$$g_i(x_i, 1 - x_i) = mx_i - (r_i/2)(x_i\sigma)^2.$$

Simple calculations reveal that

$$s^i = \left\{(x_i, x_j) \in S \mid x_i = \underset{z \in [0,1]}{\operatorname{argmin}} \left[z - \frac{m^2}{r_i\sigma^2}\right]^2\right\}$$

$$S^* = \{(x_1, x_2) \in S \mid x_1 = r_2/(r_1 + r_2)\}.$$

Now, to begin, suppose that the values of m, σ, r_1, and r_2 are such that $s^1 = (1,0)$ and $s^2 = (0,1)$; that is, both bidders' certainty equivalent gross payoffs are monotonically increasing in the share they obtain. For these cases, it is easy to see that the auctioneer does worse in the share auction than in a unit auction. Suppose that $0 \neq r_1 \leqq r_2$. Then his revenues in a unit auction are exactly $g_2(0,1)$. In a share auction, on the other hand, he receives

$$g_1(s^1) + g_2(s^2) - G_{\$}(S^*) = g_2(0,1) + [g_1(1,0) - G_{\$}(S^*)],$$

which is strictly lower, since $(1,0) \notin S^*$. Note, however, that the share auction results in an efficient allocation, while the unit auction does not.

Once an s^i involves an $x_i < 1$, however, we can also find cases where the revenue from a share auction is *higher* than that from a unit auction. The reader can easily verify that this is true in the following two sets of cases: $\{r_1 = 0, r_2 > (m/\sigma^2)\}$ and $\{r_1 = r_2 > (3/2)(m/\sigma^2)\}$.

What can we say about the net payoffs received by the two bidders in the share auction? First, since $g_i(s^i)$ is decreasing in r_i, we see that the more risk-averse bidder always receives a lower equilibrium net payoff than the less risk-averse bidder. Second, while bidder i's net payoff increases when r_i falls, it does not necessarily increase when r_j rises because this affects both $g_j(s^j)$ and $G_{\$}(S^*)$. Finally, though the more risk-averse bidder necessarily does better in a share auction than he would in a unit auction of the items (he receives zero in the unit auction), this is not the case for the less risk-averse bidder. To see this, consider the case where $\{r_1 = 0, r_2 > (m/\sigma^2)\}$. Then, since $S^* = (1,0)$ and $g_2(s^2) > g_2(1,0)$, bidder 1's net payoff in the share auction

14. We assume, as did Wilson, that resale of the shares won in the auction is not possible.

$[G_{\mathcal{J}}(1,0) - g_2(s^2)]$ is lower than his unit auction payoff of $[G_{\mathcal{J}}(1,0) - g_2(1,0)]$.

B. Economic Influence

In cases where a decision-maker accepts contingent rewards or bribes offered by interested parties, it is natural to model his decision as being governed by a first-price menu auction. One might then inquire, which party has the greatest influence (and in what sense is it "greatest"), and which parties fare well in equilibrium?

Our first problem is to select a meaningful measure of influence. One candidate is the degree to which the decision-maker's choice respects the particular party's preferences. When utility is perfectly transferable (as in our model), Theorem 2 establishes that, regardless of payoffs, no interested party will be more successful (in this sense) than any other, since the joint payoff-maximizing action is always chosen.

Yet perfectly transferable utility may abstract from the features of primary interest in many situations of economic influence. A particular party might, for example, be in a position to confer some great boon upon the decision-maker at relatively little cost to himself. The simple case of linearly transferable utility is easily handled within our framework. Suppose as before that each interested party i receives gross payoffs $g_i(s)$, and selects a set of offers $f_i(s)$. However, we now allow the effectiveness of this compensation to differ between players. Specifically, we shall assume that the decision-maker maximizes

$$\sum_{i=1}^{M} \alpha_i f_i(s).$$

It is easy to see that this game is strategically equivalent to a game of perfectly transferable utility, where interested party i receives gross payoffs $\alpha_i g_i(s)$. Theorems 2 and 3 then imply that the truthful, coalition-proof Nash equilibria will involve the decision-maker selecting

$$\underset{s \in S}{\operatorname{argmax}} \sum_{i=1}^{M} \alpha_i g_i(s).$$

In other words, the decision-maker selects a Pareto-efficient action (defined in terms of gross payoffs, with no transfers permitted), where the corresponding weights are simply the efficiency parameters of interested parties' compensation technologies. Parties who can compensate agents at low cost will have more influence, yet the action chosen will nevertheless remain Pareto efficient.

Next, how well does interested party i fare in equilibrium? The definition of $E_\Gamma(S^*)$, combined with our results, implies that subgroup J's equilibrium payoffs depend only upon the maximum joint payoff, and upon $\bar{J}$'s maximum payoff. Thus, no change in i's payoff that leaves the maximum joint payoff unaffected will alter i's equilibrium net payoff. In particular, increases in i's valuation of his most preferred action does not in general provide him with more bargaining leverage—it only hurts his opponents. In addition, notice that net payoffs are completely insensitive to the distribution of gross payoffs at the equilibrium action (so long as changes in the distribution do not affect the identities of S^J)—surprisingly, interested parties who like the equilibrium action most do not necessarily fare the best.

Finally, we have the following interesting, and intuitive, conclusion concerning the auctioneer's equilibrium compensation: his payoff rises with the level of conflict between the bidders. In the case of two interested parties and no inherent decision-maker preferences, for example, the decision-maker's equilibrium payoff, $g_A(s^A) + g_B(s^B) - G_{\mathcal{J}}(S^*)$, increases when the preference of either interested party for his favorite action becomes stronger relative to the joint maximizing action. In a game of pure division, for example, the decision-maker extracts all of the surplus [since $g_j(s^j) = G_{\mathcal{J}}(S^*)$ for $j = A,B$].

C. Delegated Common Agency

The task of modeling situations in which several parties voluntarily bestow the right to make certain decisions upon a single, common agent is quite complex. In particular, the determination of interested parties, agent action sets, and the market environment is endogenous to the process of agency formation. When an agent turns down an offered contract, the rejected principal in effect takes his choice set with him, perhaps hiring another agent, or making decisions independently.

Nevertheless, our results may have some provocative implications for situations of delegated common agency. Here, we ignore the process of delegation, and consider an agent with some well-specified clientele—an environment that is, effectively, a situation of economic influence. Within this admittedly oversimplified context, there are three important conclusions to be drawn from our model.

First, common marketing agents and brokers may serve as facilitating devices for collusion. Delegated actions are selected so as to maximize total profits for all principals. In cases where there is competition between agents, any profits extracted by the agent are presumably passed back to the principals in some form, so there is a clear incentive for competitors to choose a common agent.

Second, despite this collusive result, there is every appearance of competition. Principals both choose their agents and select their reward schedules noncooperatively. There is no need for any explicit agreement between the principals, nor must the agent bind himself to specifically act in their joint interest. Collusion arises from strategic interaction.

Third, the model provides one possible explanation of why common sales agents are usually compensated on the basis of outcomes, rather than receiving fee-for-service payment. Technically, we have allowed each principal to condition compensation on the action that the agent takes for *every* other principal. One might think that this smacks of collusion. Recall, however, that in a truthful equilibrium a principal pays rewards based only on his own gross payoffs. Thus, the appearance of noncompetitive behavior is avoided by conditioning compensation only on personally relevant outcomes (such as sales) rather than upon actions. Fee-for-service would not permit such implicit conditioning of compensation on actions taken by the agent for others. Thus, while some have explained outcome-based compensation as arising due to asymmetric information, another possible explanation is available: such compensation facilitates lawful collusion.

To take a specific example, suppose that the principals are oligopolists producing imperfectly substitutable products in a constant variable costs industry. Each firm chooses its price p_i and marketing level m_i. Sales are then given by $x_i = s_i(p,m)$ where p and m are the vectors of prices and marketing efforts chosen by all firms. A Nash Equilibrium in the absence of agency consists of vectors p^* and m^* such that, for all i (p_i^*, m_i^*) maximizes

$$s_i(p_i,p_{-i}^*, m_i,m_{-i}^*) \cdot (p_i - c_i) - m_i.$$

Now suppose that these firms hire a common marketing agent. In equilibrium, marketing effort will be chosen cooperatively, subject to a noncooperative choice of price. The cooperative levels of marketing efforts will be implemented by truthful strategies. What do these look like? Recall that each firm has transferred the marketing decision, and with it the cost m_i, to the agent. Given the noncooperative equilibrium choice of prices, its profits are now given by $s_i(p^*,m) \cdot (p_i^* - c_i)$.[15] Thus, with a truthful strategy, for each additional unit sold, principal i pays the agent $(p_i^* - c_i)$. This is precisely a *commission* schedule, where the commission fee is set equal to the excess of price over marginal cost. Thus, commission compensation arises endogenously and has the effect of facilitating complete collusion in marketing. Finally, recall that although the agent receives the full surplus $(p_i^* - c_i)$ from selling each unit, competition presumably drives his profits to zero—the surplus is returned to the principals in the form of franchise fees (or "complementary services").

APPENDIX

The following result provides a useful characterization of $E_\Gamma(s)$.

LEMMA A.1. $n \in E_\Gamma(s)$ if and only if $n \in \Pi_\Gamma(s)$ and for all i there exists $J \subseteq \mathcal{J}$, $i \in J$ such that

$$N_J = [G_{\mathcal{J}}(s) - d(s)] - [G_{\bar{J}}(s^{\bar{J}}) - d(s^{\bar{J}})],$$

where $s^{\bar{J}} \in S^{\bar{J}}$, $N_J = \sum_{i \in J} n_i$.

Proof. (i) Assume that $n \in E_\Gamma(s)$. If for all J with $i \in J$ we have $N_J < [G_{\mathcal{J}}(s) - d(s)] - [G_{\bar{J}}(s^{\bar{J}}) - d(s^{\bar{J}})]$, then it would be possible to increase n_i slightly without moving outside of $\Pi_\Gamma(s)$—a contradiction.

(ii) Assume that $n \in \Pi_\Gamma(s)$, but $n \notin E_\Gamma(s)$. Then there exists $\tilde{n} \in E_\Gamma(s)$ with $\tilde{n} \geqq n$. Without loss of generality, assume that $\tilde{n}_1 > n_1$. Consider $\hat{n} = (\tilde{n}_1, n_2, \ldots, n_M)$. Since $\tilde{n} \geqq \hat{n}$, $\hat{n} \in \Pi_\Gamma(s)$. But this cannot be unless for all $J \subseteq \mathcal{J}$ with $1 \in J$ we have $N_J < [G_{\mathcal{J}}(s) - d(s)] - [G^{\bar{J}}(s^{\bar{J}}) - d(s^{\bar{J}})]$.

Q.E.D.

We now prove our central results concerning coalition-proof equilibria.

Proof of Theorem 2. First, we show that in any truthful equilibrium, the decision-maker selects an action in S^*. Assume that

15. Note that prices are not driven down to costs due to our assumption that the goods are imperfect substitutes.

this was not the case and let $s^0 \in S^*$ be the agent's choice. Then, since strategies are truthful, we have for $s^* \in S^*$:

$$f_i(s^*) \geqq g_i(s^*) - [g_i(s^0) - f_i(s^0)].$$

Summing,

$$F_{\mathcal{I}}(s^*) \geqq G_{\mathcal{I}}(s^*) - G_{\mathcal{I}}(s^0) + F_{\mathcal{I}}(s^0)$$

so

$$F_{\mathcal{I}}(s^*) - d(s^*) \geqq \{[G_{\mathcal{I}}(s^*) - d(s^*)] - [G_{\mathcal{I}}(s^0) - d(s^0)]\} + [F_{\mathcal{I}}(s^0) - d(s^0)] > F_{\mathcal{I}}(s^0) - d(s^0),$$

a contradiction.

Next we argue that if $[\{f_i\}_{i\in\mathcal{I}}, s^*]$ is a truthful equilibrium, then for all $J \subseteq \mathcal{I}$,

$$N_J(s^*) \leqq [G_{\mathcal{I}}(s^*) - d(s^*)] - [G_{\bar{J}}(s^{\bar{J}}) - d(s^{\bar{J}})]$$

(i.e., the payoffs are in $\Pi_\Gamma(s^*)$). We see this as follows:

$$F_{\mathcal{I}}(s^*) - d(s^*) \geqq F_{\mathcal{I}}(s^{\bar{J}}) - d(s^{\bar{J}})$$

so

$$\begin{aligned} F_J(s^*) + F_{\bar{J}}(s^*) - d(s^*) &\geqq F_J(s^{\bar{J}}) + F_{\bar{J}}(s^{\bar{J}}) - d(s^{\bar{J}}) \\ &\geqq F_{\bar{J}}(s^{\bar{J}}) - d(s^{\bar{J}}) \\ &\geqq [G_{\bar{J}}(s^{\bar{J}}) - G_{\bar{J}}(s^*) + F_{\bar{J}}(s^*)] - d(s^{\bar{J}}), \end{aligned}$$

where this last inequality comes from the fact that, since strategies are truthful, for each $j \in \bar{J}$, $f_j(s^{\bar{J}}) \geqq g_j(s^{\bar{J}}) - g_j(s^*) + f_j(s^*)$. Rearranging this last inequality yields the desired result.

Now we argue that the payoffs are actually in $E_\Gamma(s^*)$. Choose some i. In an equilibrium, there exists $s_i \in I^*(\{f_i\}_{i\in\mathcal{I}})$ such that $f_i(s_i) = 0$. Take $J = \{j \in \mathcal{I} \mid f_j(s_i) = 0\}$; this will obviously include i. Since both s_i and s^* belong to $I^*(\{f_i\}_{i\in\mathcal{I}})$, we know that

$$F_{\mathcal{I}}(s^*) - d(s^*) = F_{\mathcal{I}}(s_i) - d(s_i)$$

so

$$F_J(s^*) + F_{\bar{J}}(s^*) - d(s^*) = F_J(s_i) + F_{\bar{J}}(s_i) - d(s_i),$$

which yields, by the definitions of J and $\bar{J}$ (note that since strategies are truthful, for $j \in \bar{J}$ we have $f_j(s_i) = g_j(s_i) - g_j(s^*) + f_j(s^*)$),

$$F_J(s^*) + F_{\bar{J}}(s^*) - d(s^*) = [G_{\bar{J}}(s_i) - G_{\bar{J}}(s^*) + F_{\bar{J}}(s^*)] - d(s_i)$$

or, rearranging,

$$N_J(s^*) = [G_{\mathcal{I}}(s^*) - d(s^*)] - [G_{\bar{J}}(s_i) - d(s_i)].$$

Now, we can show that $s_i \in S^{\bar{J}}$. Suppose not. Then there exists $\tilde{s}$ such that

$$G_{\bar{J}}(\tilde{s}) - d(\tilde{s}) > G_{\bar{J}}(s_i) - d(s_i).$$

Since $G_{\bar{J}}(s) \leqq G_{\bar{J}}(s^*) - F_{\bar{J}}(s^*) + F_{\bar{J}}(s)$ for all $s \in S$ and with equality for $s = s_i$, we have

$$G_{\bar{J}}(s^*) - F_{\bar{J}}(s^*) + F_{\bar{J}}(\tilde{s}) - d(\tilde{s}) > G_{\bar{J}}(s^*) - F_{\bar{J}}(s^*) + F_{\bar{J}}(s_i) - d(s_i)$$

or,

$$F_{\bar{J}}(\tilde{s}) - d(\tilde{s}) > F_{\bar{J}}(s_i) - d(s_i).$$

Since $F_J(\tilde{s}) \geqq F_J(s_i)$, we have a contradiction to $s_i \in I^*[\{f_i\}_{i \in \mathcal{I}}]$. But this establishes that for each i, there exists $J \subseteq \mathcal{I}$ with $i \in J$ such that the net payoff constraint for J binds. By Lemma A.1, net payoffs must lie in $E_\Gamma(s^*)$. This establishes the first part of the theorem.

Now choose any $n \in E_\Gamma(s^*)$ and consider the truthful strategies given by $f_i(s) = \max[g_i(s) - n_i, 0]$ for all $s \in S$, $i \in \mathcal{I}$. Note that since $n_i \leqq [G_{\mathcal{I}}(s^*) - d(s^*)] - [G_{-i}(s^{-i}) - d(s^{-i})]$, we have

$$\begin{aligned} g_i(s^*) - n_i &\geqq \{[G_{-i}(s^{-i}) - d(s^{-i})] - [G_{-i}(s^*) - d(s^*)]\} \\ &\geqq 0, \end{aligned}$$

since the bracketed term is positive by definition. Hence $f_i(s^*) = g_i(s^*) - n_i$, or $g_i(s^*) - f_i(s^*) = n_i$. Thus, if this is in fact a Nash Equilibrium supporting s^*, it yields the correct net payoffs.

We now check that $[\{f_i\}_{i=1}^M, s^*]$ is a Nash Equilibrium. To do this, we verify each condition in Lemma 1.

(i) $f_i(s) \geqq 0$ for all $s \in S$ by construction.

(ii) Suppose that the agent receives higher total utility at some $s \neq s^*$. First, note that $F_{\mathcal{I}}(s^*) - d(s^*) = G_{\mathcal{I}}(s^*) - N_{\mathcal{I}} - d(s^*)$. Now, divide the principals into two groups: for $i \in J$, $f_i(s) = g_i(s) - n_i$; for $i \in \bar{J}$, $f_i(s) = 0$. Then we know that

$$G_J(s) - N_J - d(s) > G_{\mathcal{I}}(s^*) - d(s^*) - N_{\mathcal{I}}$$

Collecting terms,

$$N_{\bar{J}} > [G_{\mathcal{I}}(s^*) - d(s^*)] - [G_J(s) - d(s)].$$

Since for $s^J \in S^J$

$$G_J(s^J) - d(s^J) \geqq G_J(s) - d(s),$$

we have

$$N_{\bar{J}} > [G_{\mathcal{I}}(s^*) - d(s^*)] - [G_J(s^J) - d(s^J)],$$

which contradicts that $n \in \Pi_\Gamma(s^*)$.

$$\text{(iii) } f_i(s) \geqq g_i(s) - n_i = g_i(s) - g_i(s^*) + f_i(s^*),$$

so

$$g_i(s^*) - g_i(s) \geqq f_i(s^*) - f_i(s).$$

Since

$$[F_{\mathcal{J}}(s) - d(s)] - [F_{\mathcal{J}}(s^*) - d(s^*)] \leqq 0,$$

we have

$$g_i(s^*) - g_i(s) \geqq [F_{\mathcal{J}}(s) - d(s)] - [F_{\mathcal{J}}(s^*) - d(s^*)] + f_i(s^*) - f_i(s) = [F_{-i}(s) - d(s)] - [F_{-i}(s^*) - d(s^*)].$$

To prove property (iv), we shall need the following result.

LEMMA A.2. Assume that $f_i(s) = \max[g_i(s) - n_i, 0]$ for all $s \in S$ and all $i \in \mathcal{J}$, and that $N_J = [G_{\mathcal{J}}(s^*) - d(s^*)] - [G_{\bar{J}}(s^{\bar{J}}) - d(s^{\bar{J}})]$. Then

(a) $F_{\mathcal{J}}(s^{\bar{J}}) - d(s^{\bar{J}}) = F_{\mathcal{J}}(s^*) - d(s^*)$

(b) For all $i \in J$, $f_i(s^{\bar{J}}) = 0$

(c) For all $i \in \bar{J}$, $f_i(s^{\bar{J}}) = g_i(s^{\bar{J}}) - n_i$.

Proof: We know that

$$G_J(s^*) - F_J(s^*) = [G_{\mathcal{J}}(s^*) - d(s^*)] - [G_{\bar{J}}(s^{\bar{J}}) - d(s^{\bar{J}})].$$

We also know that

$$F_J(s^{\bar{J}}) \geqq 0 \tag{A.1}$$

so

$$F_J(s^*) \leqq [G_{\bar{J}}(s^{\bar{J}}) - d(s^{\bar{J}})] - [G_{\bar{J}}(s^*) - d(s^*)] + F_J(s^{\bar{J}}). \tag{A.2}$$

Further, we know that

$$F_{\bar{J}}(s^{\bar{J}}) \geqq G_{\bar{J}}(s^{\bar{J}}) - N_{\bar{J}} \tag{A.3}$$

which, substituting for $N_{\bar{J}}$ and rearranging, yields

$$F_{\bar{J}}(s^*) \leqq [G_{\bar{J}}(s^*) - G_{\bar{J}}(s^{\bar{J}})] + F_{\bar{J}}(s^{\bar{J}}). \tag{A.4}$$

Summing (A.4) and (A.2), we find (rearranging) that

$$F_{\mathcal{J}}(s^*) - d(s^*) \leqq F_{\mathcal{J}}(s^{\bar{J}}) - d(s^{\bar{J}}). \tag{A.5}$$

But we know from part (ii) that this cannot be $<$, so we must have equality. This establishes part (a). Parts (b) and (c) follow immediately—if either (A.1) or (A.3) holds with strict inequality, then (A.5) must hold with strict inequality, contradicting (ii).

Q.E.D.

Now we return to the main theorem.

(iv) Since $n \in E_\Gamma(s^*)$, we know (Lemma A.1) that for all i, there exists $J \subseteq \mathcal{I}$ with $i \in J$ such that

$$N_J = [G_{\mathcal{I}}(s^*) - d(s^*)] - [G_{\bar{J}}(s^{\bar{J}}) - d(s^{\bar{J}})].$$

But then, by Lemma A.2, $F_{\mathcal{I}}(s^{\bar{J}}) - d(s^{\bar{J}}) = F_{\mathcal{I}}(s^*) - d(s^*)$, and $f_i(s^{\bar{J}}) = 0$. Thus, any $n \in E_\Gamma(s^*)$ can be supported in a Truthful Nash Equilibrium.

Q.E.D.

Proof of Theorem 3. The argument is by induction. The case of $M = 1$ is trivial. Now, assume that the theorem is true for $M = 1, \ldots, K - 1$ and consider the case of $M = K \geqq 2$.

We first argue that any self-enforcing set of offers by the K interested parties that results in decision s^0, must offer net payoffs to the interested parties that lie in $\Pi_\Gamma(s^0)$. By the definition of a self-enforcing strategy profile, and the fact that the theorem is true for $M < K$, we see that for any nonempty subgroup $J \subset \mathcal{I}$,

$$s^0 \in \arg\max_{s \in S} G_{\bar{J}}(s) + F_J(s) - d(s).$$

From this, we find that for all $s \in S$, and $J \subset \mathcal{I}$,

$$F_J(s^0) - F_J(s) \geqq [G_{\bar{J}}(s) - d(s)] - [G_{\bar{J}}(s^0) - d(s^0)].$$

Since $F_J(s) \geqslant 0$, we see that

$$G_J(s^0) - F_J(s^0) \leqq [G_{\mathcal{I}}(s^0) - d(s^0)] - [G_{\bar{J}}(s) - d(s)].$$

In particular, setting $s = s^{\bar{J}}$ and substituting $N_J = G_J(s^0) - F_J(s^0)$ yields

$$N_J \leqq [G_{\mathcal{I}}(s^0) - d(s^0)] - [G_{\bar{J}}(s^{\bar{J}}) - d(s^{\bar{J}})] \text{ for all } J \subset \mathcal{I}. \tag{A.6}$$

This must be true for all $J \subset \mathcal{I}$. Furthermore, it is obvious that this condition must hold for $J = \mathcal{I}$, since (choosing $\underline{s} \in S^{\bar{J}}$),

$$F_{\mathcal{I}}(s^0) - d(s^0) \geqq - d(\underline{s}). \tag{A.7}$$

(otherwise the decision-maker would select $\underline{s}$). From (A.6) and (A.7), we conclude that any self-enforcing set of offers that result in the decision-maker choosing action s^0, must yield net payoffs that lie in $\Pi_\Gamma(s^0)$.

Next, note that since $G_{\mathcal{I}}(s^*) - d(s^*) > G_{\mathcal{I}}(s^0) - d(s^0)$ where $s^* \in S^*$ and $s^0 \notin S^*$, any point in $\Pi_\Gamma(s^0)$ is strictly dominated by a point in $E_\Gamma(S^*)$. Therefore, by establishing that any net payoff vector in $E_\Gamma(S^*)$ can be supported by a Coalition-Proof Nash Equilibrium, we shall have established that all Coalition-Proof Nash Equilibria support net payoff vectors in $E_\Gamma(S^*)$. We now turn to demonstrating this fact.

We showed in Theorem 2 that any net payoff vector in E_Γ (S^*) can be supported by a Truthful Nash Equilibrium. If we can show that any such strategy is also self-enforcing, then, since all self-enforcing strategy profiles offer net payoffs in $\Pi_\Gamma(S^*)$, these Truthful Nash Equilibrium strategies will also constitute a Coalition-Proof Nash Equilibrium. Note, in addition, that by Theorem 2 this implies that *all* Truthful Nash Equilibrium are coalition-proof.

Now, consider an $n \in E_\Gamma(S^*)$ and the truthful set of offers:

$$f_i(s) = \max[g_i(s) - n_i, 0] \text{ for all } s \in S \text{ and } i \in \mathcal{J}.$$

We have already seen above that these strategies constitute a Truthful Nash Equilibrium in which the decision-maker chooses $s^* \in S^*$. We proceed in two steps. First, we must show that for all J, s* is the joint maximizing action for $\Gamma/\{f_i\}_{i \in \bar{J}}$. Specifically, we must verify that s^* maximizes $G_J(s) - d(s) + F_{\bar{J}}(s)$. We know that

$$F_J(s) \geqq G_J(s) - N_J = G_J(s) - G_J(s^*) + F_J(s^*)$$

or, rearranging,

$$G_J(s^*) - F_J(s^*) \geqq G_J(s) - F_J(s).$$

Further, we know that

$$F_{\mathcal{J}}(s^*) - d(s^*) \geqq F_{\mathcal{J}}(s) - d(s).$$

Summing these inequalities, we get

$$G_J(s^*) + F_{\bar{J}}(s^*) - d(s^*) \geqq G_J(s) + F_{\bar{J}}(s) - d(s)$$

as desired.

The second step is to show that for all $J \subset \mathcal{J}$, $(n_i)_{i \in J} \in E_{\Gamma/\{f_j\}_{j \in \bar{J}}}(s^*)$. If this is true then, by the induction assumption, no subgroup can make a credible (self-enforcing) counterproposal that weakly benefits all of its members; hence the equilibrium is self-enforcing.

Choose any $i \in J$, and T with $i \in T$ such that

$$N_T = [G_{\mathcal{J}}(s^*) - d(s^*)] - [G_{\bar{T}}(s^{\bar{T}}) - d(s^{\bar{T}})].$$

Noting that

$$N_T = N_{T \cap J} + N_{T \cap \bar{J}}$$

and

$$N_{T \cap \bar{J}} = G_{T \cap \bar{J}}(s^*) - F_{T \cap \bar{J}}(s^*),$$

we see that

$$\begin{aligned} N_{T\cap J} = {} & [G_J(s^*) + F_{\overline{J}}(s^*) - d(s^*)] \\ & - [G_{\overline{T}\cap J}(s^{\overline{T}}) + F_{\overline{J}}(s^{\overline{T}}) - d(s^{\overline{T}})] \\ & + \{[G_{\overline{T}\cap \overline{J}}(s^*) - F_{\overline{T}\cap \overline{J}}(s^*)] - [G_{\overline{T}\cap \overline{J}}(s^{\overline{T}}) - F_{\overline{T}\cap \overline{J}}(s^{\overline{T}})]\} \\ & + F_{T\cap \overline{J}}(s^{\overline{T}}). \end{aligned}$$

By Lemma A.2(c), the term in curly brackets is identically zero, since $(\overline{T} \cap \overline{J}) \subseteq \overline{T}$. Also, by Lemma A.2(b), the last term is zero since $(T \cap \overline{J}) \subseteq T$. So,

$$\text{(A.8)} \quad \begin{aligned} N_{T\cap J} = {} & [G_J(s^*) + F_{\overline{J}}(s^*) - d(s^*)] \\ & - [G_{\overline{T}\cap J}(s^{\overline{T}}) + F_{\overline{J}}(s^{\overline{J}}) - d(s^{\overline{T}})]. \end{aligned}$$

Now we argue that $s^{\overline{T}}$ maximizes $G_{\overline{T}\cap J}(s) + F_{\overline{J}}(s) - d(s)$. Notice first that

$$\begin{aligned} F_{\overline{J}}(s^{\overline{T}}) &= F_{\mathcal{J}}(s^{\overline{T}}) - F_J(s^{\overline{T}}) \\ &= F_{\mathcal{J}}(s^{\overline{T}}) - F_{\overline{T}\cap J}(s^{\overline{T}}) - F_{T\cap J}(s^{\overline{T}}) \\ &= F_{\mathcal{J}}(s^{\overline{T}}) - G_{\overline{T}\cap J}(s^{\overline{T}}) + N_{\overline{T}\cap J}. \end{aligned}$$

Now suppose that there exists s such that

$$G_{\overline{T}\cap J}(s) + F_{\overline{J}}(s) - d(s) > G_{\overline{T}\cap J}(s^{\overline{T}}) + F_{\overline{J}}(s^{\overline{T}}) - d(s^{\overline{T}}).$$

Substituting, we know that

$$\begin{aligned} G_{\overline{T}\cap J}(s^{\overline{T}}) + F_{\overline{J}}(s^{\overline{T}}) - d(s^{\overline{T}}) &= N_{\overline{T}\cap J} + [F_{\mathcal{J}}(s^{\overline{T}}) - d(s^{\overline{T}})] \\ &= N_{\overline{T}\cap J} + [F_{\mathcal{J}}(s^*) - d(s^*)] \end{aligned}$$

by Lemma A.2 (a). Further, since $N_{\overline{T}\cap J} \geqq G_{\overline{T}\cap J}(s) - F_{\overline{T}\cap J}(s)$, and $F_{T\cap J}(s) \geqq 0$, we have

$$\begin{aligned} & G_{\overline{T}\cap J}(s) + F_{\overline{J}}(s) - d(s) \\ & \qquad > G_{\overline{T}\cap J}(s) - F_{\overline{T}\cap J}(s) - F_{T\cap J}(s) + [F_{\mathcal{J}}(s^*) - d(s^*)]. \end{aligned}$$

Collecting terms,

$$F_{\mathcal{J}}(s) - d(s) > F_{\mathcal{J}}(s^*) - d(s^*),$$

which is a contradiction.

Thus, the interpretation of (A.8) is that in $\Gamma/\{f_j\}_{j\in\overline{J}}$, for each $i \in J$ there is some constraint in $\Pi_{\Gamma/\{f_i\}_{i\in\overline{J}}}(s^*)$ involving n_i, which binds. By Lemma A.1, we then know that $\{n_i\}_{i\in J} \in E_{\Gamma/\{f_i\}_{i\in\overline{J}}}(s^*)$. Thus, using the induction assumption, the equilibrium is self-enforcing. Since it is on the efficient frontier of self-enforcing equilibria, it is coalition-proof. Apply induction.

Q.E.D.

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